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THEORETICAL FOUNDATIONS OF FORECASTING THE RESOURCE OF HYDRAULIC STRUCTURES

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Abstract. It is established that in terms of the discrete Markov process, the problem is reduced to the search for unconditional probabilities of the system S at an arbitrary step k in state S_i, that is, obtaining a transition probabilities matrix. In this formulation, the model is used for assessing the technical condition of the element; assessing the level of safety of operation of structural elements; ranking elements according to the need for repairs, reconstruction or replacement; in strategic planning of repair or reconstruction costs in conditions of limited funding and forecasting the remaining resource of elements.

It is established that the theoretical basis of the study, which aims to predict the resource of hydraulic structures in operation, is the Markov theory of random processes. For a mathematical description of the process of element degradation, the most successful is the mathematical apparatus of the Markov random processes.

Determination of the failure intensity parameter is the dominant feature of the Markov phenomenological model of damage accumulation to hydraulic structures' elements. The only parameter of lifecycle management is the failure rate λ. In the model under consideration, the parameterλ is determined based on the initial conditions for an individual element obtained from the survey results.

Because the parameter λ is determined for an individual element and must be specified each time after the next survey, the accuracy of the model will increase. The proposed model is integral. It does not contain an explicit theoretical apparatus for a material-sensitive element, its static scheme, construction technology, environmental conditions, etc. On the other hand, all these factors and many other secondary ones are taken into account in the model at the moment the state of the element is determined using classification tables containing physical and mechanical signs of degradation.

In the theory of structures, the statistical approach to formulating the transition matrix is widespread and is based on historical data from the structure operation system. It is believed that the transition matrix based on the data of the operating system is a more realistic basis for predicting the processes of structures degradation. A large number of foreign studies are devoted to the practical application of the transition matrix based on statistical data, which consider the features of transition matrices related to the bridge operation system in different countries. In this formulation, each element of the transition probability matrix P is the probability that the system in the state will move to state j in one step (i.e., in one year). At the same time, it is considered that there are no operational interventions, so the sub-diagonal elements are zero. As before, the sum of elements of the same line is 1 and the element p_{jj} = 1 because state *j* is absorbing.

For the implementation algorithm of the Markov chain model for forecasting the technical condition of hydraulic structures in general, the initial data are: statistical data of the distribution of structures by the state at the time of the forecast, the rating assessment of the structure is calculated by an expert according to the scale and the forecast time in years.

It is established that the degradation properties of structural designs are described by two parameters: the degradation criterion and the failure rate. Any factor of the stress-strain state can be taken as a degradation criterion: reliability, internal forces, or deformations. The degradation criterion can be an arbitrary rating assessment. In our case, the reliability of the element is taken as the degradation criterion, as the most general factor of the stress-strain state.

Keywords: hydraulic structures, structural degradation of structures, Markov model, service life.

Introduction. The classical a priori formulation of the life cycle of a hydraulic structure here receives a rigorous scientific justification regarding resource – the structures are associated with universal models of describing the phenomenological degradation processes of hydraulic structures' elements by random functions of the Markov type. The central scientific idea of this approach is *a new paradigm of the theory of structures is to establish the relationship between the equations of boundary States and the variable of operating time.*

The problem of estimating and forecasting the resource, as a category of durability, is relevant not only for the latest hydrotechnical structures but also has an independent extraordinary weight as a factor of the state strategy for managing and preventing man-made risks. All countries face this problem, but the problem is becoming particularly significant due to several unfavourable reasons for Ukraine today. Among them, there is the complicated economic and financial situation in the country that provokes an increase in the rate of elements' degradation due to a chronic decrease in funding for maintaining the technical condition of hydraulic structures.

We should admit that the number of physically outdated structures is rapidly growing in the infrastructure now. Under these conditions, for trouble-free operation and extension of the service life of structures, new scientific approaches are needed to assess and predict the technical state of the hydraulic structures' elements at all stages of the life cycle and establish scientifically based service life. Such algorithms that provide quantitative criteria for the level of reliability in the time function and, accordingly, the ability to predict the resource of hydraulic structures' elements are considered in this paper.

The problem and its relevance. For a long time, the problem of the durability of hydraulic structures was the subject of attention exclusively to academic circles. Forecasting the resource of the hydraulic structures' elements while designing and operating was not paid due attention in the theory of structures. The longevity control device has always been primitive and the least studied. Indeed, there are no levers to control durability in the design apparatus of hydraulic structures. Their life cycle term is assigned directively, the calculated dependencies do not have variable time, and the durability problem is entirely in the designer's experience and intuition. On the other hand, the problem of resource assessment was and is the most significant in socio-economic terms. It is obvious that under these conditions, the models are aimed at evaluating and predicting the durability of hydraulic structures with practical implementation and meet the interests of society and state policy in man-made and economic security.

Analysis of recent research and publications. In scientific works [1–20], applied research aimed at the theoretical foundation's development to estimate and forecast the life cycle of construction structures and the practical apparatus to manage their resources is widely used.

In the most general form, the modern formulation of the durability problem is given in the documents of the Joint Committee on Structural Safety in the work "The typical model" ("Probabilistic Model Code,1996") [15-17] and in the monograph by Robert E. Melchers [21], as the probability of reaching the limit state per time. To do this, enter a time-dependent limit state function:

$$g(\mathbf{X},t) = R(\mathbf{X},t) - Q(\mathbf{X},t), \tag{1}$$

where $R(\mathbf{X},t)$ – generalized element resistance; $Q(\mathbf{X},t)$ – generalized loading effect; $\mathbf{X}$ – vector of basic variables; t – time variable.

The reliability function itself, how is the probability of reaching the limit state over time t it has the form:

$$P(t) = \text{Prob}[\min g(\mathbf{X}(\tau);\tau) < 0 \text{ for } 0 < \tau < t] \tag{2}$$

or in terms of the limit state function:

$$P(t) = \text{Prob}[R(\mathbf{X},t) - Q(\boldsymbol{X},t) < 0]. \tag{3}$$

Thus, by dependencies (2) and (3), durability is formulated as a concept functionally related to reliability. Maximum value t, which satisfies equation (1-3) is the resource of the element.

The theoretical basis of the study, which aims to predict the resource of construction structures' elements in operation, is Markov's theory of random processes. Markov's theory is a process whose evolution only depends on a fixed current state over time. As it turned out in the last 15-20 years, the mathematical apparatus of random Markov processes is the most successful for the mathematical description of construction *structures' elements' degradation process*.

The degradation of elements in operation will be considered a flow of failures. In our case, the flow is considered a hierarchy of failures, which is physically a manifestation of the degradation of elements under the influence of loads and the environment. The stationary simplest Poisson-type flow is considered [1, 2].

The research objective is the theoretical foundations to predict the resource of hydraulic structures.

Materials and methods of the research. A mathematical model of a random process with continuous time and discrete states, the graph of which is linear, is called the Markov chain [1, 3, 4]. The Markov chain is described using *probabilities of states*. Let's denote the probability of states k of a chain pitch in this way:

$$p_i(k) = P(S_i^{(k)});\; p_{i+1}^{(k)} = P(S_{i+1}^{(k)}), \tag{4}$$

where k – a step number, $k = 1,2,..., n-1$; n – a status number, $n = 1, 2, ...$

For an arbitrary step of the Markov chain, there are certain probabilities of transition from one discrete state to another. *The probability of transition* or *the transition probability* at step k from the state S_i in the state S_j is called the conditional probability that the system $\boldsymbol{S}$ after step k will be in S_j provided that immediately before that it was in state S_i.

Let's denote p_{ij} as the probability of switching from state i to state j in one step. In this case, we will assume that time $t_i < t_j$. It is convenient to record the probabilities of transition from state i into state j in the form of a square matrix. So, for example, when $n = 5$ (n – the number of states) we will have:

$$\mathbf{P} = \begin{bmatrix} p_1 & 1-p_1 & 0 & 0 & 0 \\ 0 & p_2 & 1-p_2 & 0 & 0 \\ 0 & 0 & p_3 & 1-p_3 & 0 \\ 0 & 0 & 0 & p_4 & 1-p_4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

Matrix $\mathbf{P}$ is called *a homogeneous transition matrix* (transient probabilities). The sum of the transition probabilities of an arbitrary string is equal to one:

$$\sum_{j=1}^{n} p_{\text{ij}} = 1. \tag{6}$$

A vector of initial probabilities is added to the transition matrix, which sets the distribution of absolute probabilities at the beginning of the process:

$$\mathbf{p_0} = [p_1, p_2, ..., p_n]^T. \tag{7}$$

The Markov chain is fully characterized by matrix $\mathbf{P}$ with the initial probability vector $\mathbf{p_0}$.

By the known transition matrix $\mathbf{P}$ and the vector of initial probabilities $\mathbf{p_0}$, the absolute *probabilities* of the system's states after the fixed number of transition steps can be defined. So denoting $p_i^{(n)}$ – the absolute probabilities of the system's states after n the expression of the absolute probability of the system in one step is:

$$p_{\text{i}}^{(1)} = p_1^{(0)} p_{1\text{j}} + p_2^{(0)} p_{2\text{j}} + \ldots + p_{\text{n}}^{(0)} p_{\text{nj}} = \sum_i p_{\text{i}}^{(0)} p_{\text{ij}} , \tag{8}$$

where $p_i^{(1)}$ – the absolute probability of the system moving from state to state in one step; $p_i^{(0)}$ – initial probability, a component of the vector $\mathbf{R_0}$; p_{ij} – transient probabilities of the system.

Similarly to (8), by induction, we can show that:

$$p_{\text{j}}^{(n)} = \sum_i p_i^{(0)} p_{ij}^{(n)} \tag{9}$$

where $p_{ij}^{(n)}$ – n-step transition probability, determined by recurrent formula [1, 4]:

$$p_{\text{i}}^{(k)} = \sum_{i=1}^{n} p_{\text{j}}^{(k-1)} p_{\text{ji}} \quad i = 1, 2, \ldots, n; \; j = 1, 2, \ldots, n, \tag{10}$$

where p_{ij} – transition probabilities, matrix elements $\mathbf{P}$; k – step number; n – the number of states. Formula (10) is convenient to use when at the beginning of the process only one first of the component of the vector $\mathbf{p_0}$ is known, that is, the initial probability in the first state.

Next, the task is: to find the probabilities of events of the Markov chain $p_1(t)$, $p_2(t)$,…,$p_{\text{n}}(t)$, as functions of time. We emphasize that we are now considering *a homogeneous* Markov chain, i.e. one whose transition probabilities are not a function of the step number.

Let's consider a failure chain from n events. How each of the failures is characterized is irrelevant now. Firstly, the chain must n possible states: S_1, S_2,, S_{n}, and secondly, the states are connected with a linear graph of the chain, and transitions occur only in one direction – from state i to state $i + 1$.

The probability of making a step k on which the system will switch from state S_i into state S_{i+1} is characterized by a transition probability density: $\lambda_{i,\, i+1}$. Find the function $p_1(t)$ – the probability that the element at a given time $t+\Delta t$ is in state of S_1. To do this, we will give t small increment Δt. It is necessary that within the time Δt the element did not leave a state $\boldsymbol{S_1}$. The probability of this is found as the product of the probability $p_1(t)$ by *the conditional probability* that in time Δt the element will enter state S_2. This probability is $1 - \lambda_{12}\Delta t$. Applying the probability addition rule, we get:

$$p_1(t+\Delta t) - p_1(t)(\, 1-\lambda_{12}\Delta t). \tag{11}$$

Expand the parentheses in this expression, transfer $p_1(t)$ to the left side and divide both parts of the equation by Δt, we get:

$$\frac{p_1(t+\Delta t) - p_1(t)}{\Delta t} = \lambda_{12} p_1(t) . \tag{12}$$

Next, we will direct Δt to zero and go to limit:

$$\lim_{\Delta t \to 0} \frac{p_1(t+\Delta t) - p_1(t)}{\Delta t} = -\lambda_{12} p_1(t) . \tag{13}$$

The left side of this expression is the derivative of function $p_1(t)$:

$$\frac{dp_1(t)}{dt} = -\lambda_{12} p_1(t) \tag{14}$$

Thus, a differential equation is obtained, which must satisfy function $p_1(t)$.

The differential equation for determining the function $p_2(t)$ is obtained in the same way, with the difference that at the moment of time $t+\Delta t$ we have for state S_2 two situations:

– at the moment t the element was able to S_1, and for time Δt moved to state S_2;

– or at the moment t the item was already in S_2 and over time Δt not moved to state S_3. Given that, the differential equation for $p_2(t)$ is recorded as:

$$\frac{dp_2(t)}{dt} = -\lambda_{23} p_2(t) + \lambda_{12} p_1(t) . \tag{15}$$

Further, the entire system of differential equations is compiled according to one sample:

$$\frac{dp_i(t)}{dt} = -\lambda_{i\,,i+1}\, p_i(t) + \lambda_{i-1,i}\, p_{i-1}(t), \; i = 1, 2, \ldots, n. \tag{16}$$

In general, the equations of probability states (16) are written as follows:

$$\frac{dp_{ij}(t)}{dt} = \sum_k \lambda_{ik} p_{kj}(t); \;\; i, j, k = 1, 2, \ldots, n. \tag{17}$$

These are the known Kolmogorov-Chapman equations describing the evolution of a discrete Markov process with continuous time [3-5]. In matrix form, equations (20) are:

$$\frac{d\mathbf{P}(i, t)}{dt} = \mathbf{P}(i, t) \cdot \mathbf{E}, \tag{18}$$

where **E** – state flow intensity vector.

By integrating the system of equations (18), the desired probabilities of states – the time function are obtained. The initial conditions for integration are as follows:

$$\text{at } t = 0 \;\; p_1(t)=1; \;\; p_2(\text{t}) = p_3(\text{t}), \ldots, = p_\text{n}(\text{t}) = 0. \tag{19}$$

In addition, the normalization condition is used in solving a system of differential equations:

$$\sum_{i=1}^{n} p_i(t) = 1, \tag{20}$$

which is a consequence of the fact that the events of Markov chains are incompatible and form a complete group. The solution of equations (20) is a matrix *of the transition probabilities* in the form of time-dependent variables.

By the known transition matrix elements **P,** and the initial probabilities vector $\mathbf{p_0}$ are defined by *absolute probabilities* system states *in the time function* after the fixed number of transition steps:

$$p_k(t) = \sum_{k=l-1}^{n} p_{k-1} p_{ik}(t), \tag{21}$$

where l – current status number; n – the number of discrete states in the element's lifecycle; p_k – *absolute probability* of the element in k – the discrete state; $p_\text{ik}(t)$ – transient probability k – of the discrete state.

Model of the Markov chain based on statistical data of the history of construction structures in operation. Let's consider a family of random variables $\{S_\text{tk}\}$, forming a stochastic process at the time t_k, the system can be located, and form a complete group of events. The number of system states is finite.

Hypotheses **A**. A system is defined by a set of finite states and can only be in one of them at the moment.

Б. The initial state of the system and the probability distribution of the initial state are known.

В. The stochastic process here is represented by an integral distribution function $P(t)$ for the time T_n, which proceeds until all n process events happen – Poisson distribution:

$$P(t) = 1 - P(T_n > t) = 1 - \sum_{k=0}^{n-1} \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \tag{22}$$

where λ is the process parameter failure rate (degradation rate); P_t – the probability that the element will enter the state k during the time $t < T_k$.

Г. The process object is a building. It follows from formula (22) that the states of the system at any moment t are set by their numbers k=1, 2, …, n. The transition from state S_j into state S_n exactly at k steps can occur in different ways. We consider *a special case* of the Markov chain in which movement through the states occurs only in one direction, sequentially from state j to state j + 1:

$$j < j_1 < j_2 \ldots < j_{\text{n-1}} < j_\text{n}. \tag{23}$$

In other terms, we consider the process graph in which the transition only to the neighbouring state is allowed, that is, "jumps" are excluded.

To formulate the model, let's use the time-dependent transition matrix **P**. Each element of this matrix p_{ij} is a probability that the system will change from state i to state j during *a certain period*. Then if the initial state $\mathbf{p}_0$ is known, then the future state of the system can be predicted for any arbitrary time t.

Future state vector $\mathbf{p}_t$ can be obtained by multiplying the initial state of vector $\mathbf{p}_0$ to transition matrix $\mathbf{P}$ in degree t (the number of years) [3, 4]. The initial state of the system is set by the tape-matrix $\mathbf{p}_0$ in size [1 X n]:

$$\mathbf{p}_0 = [p_1, p_2, \ldots, p_\mathrm{n}], \tag{24}$$

where p_i – probability of being in state $i = 1, 2, \ldots, n$; n – the number of discrete states. The system state vector for time t is defined as the product of transition matrix $\mathbf{P}$ on the initial state vector of the system:

$$\mathbf{p}_\mathrm{t} = \mathbf{R_0} \text{ X } \mathbf{P}^t, \tag{25}$$

where $\mathbf{P}^t$ – transition matrix $\mathbf{P}$ in degree t; $\mathbf{R_0}$ – probabilities vector in the initial state.

Next, a certain states vector is introduced $\mathbf{d}$ in size [n X 1] – vector of the fixed rating expert assessments of the structure in safe operation:

$$\mathbf{d} = [r_1, r_2, \ldots, r_n]^\mathrm{T}, \tag{26}$$

Here T – the transpose sign; r_and – ratings, real numbers, $i = 1, 2, \ldots, n$.

The technical condition of the structure for any time t is defined by the dependency:

$$D_t = \mathbf{p}_0 \text{ X } \mathbf{P}^t \text{ X } \mathbf{d}, \tag{27}$$

where D_t – rating assessment of the structure at time t – scalar; $\mathbf{p}_0$ – matrix-tape in size [1 X n] probabilities in the initial state t_0; $\mathbf{P}^t$ – transition probability matrix $\mathbf{P}$ in degree t in size [n X n].

Presentation of the main research material. Here is a hypothetical illustrative example that demonstrates the procedure for implementing the Markov model to predict the structure state based on the known transition probability matrix. For the construction: the number of fixed discrete states is 5; the initial state vector (the safe operation rating vector):

$$\mathbf{d} = [10\ 8\ 6\ 4\ 2]^\mathrm{T},$$

– Matrix-tape in size [1 X n] $\mathbf{R}_0$ probabilities in the initial state at t_0:

$$\mathbf{p}_0 = [1\ 0\ 0\ 0\ 0]:$$

– transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 0{,}939 & 0{,}061 & 0 & 0 & 0 \\ 0 & 0{,}679 & 0{,}330 & 0 & 0 \\ 0 & 0 & 0{,}549 & 0{,}451 & 0 \\ 0 & 0 & 0 & 0{,}449 & 0{,}551 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{28}$$

Find the construction rating forecast after $t = 5$ years in operation.

The solution is the technical condition of the structure expressed in a rating assessment on a scale of 10 – 2 points after 5 years in operation for any time t is defined by the dependency:

$$D_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0{,}730 & 0{,}137 & 0{,}079 & 0{,}039 & 0{,}018 \\ 0 & 0{,}144 & 0{,}240 & 0{,}266 & 0{,}374 \\ 0 & 0 & 0{,}050 & 0{,}143 & 0{,}808 \\ 0 & 0 & 0 & 0{,}018 & 0{,}982 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 8 \\ 6 \\ 4 \\ 2 \end{bmatrix} = 9{,}066$$

Formulation of the transition matrix based on statistical data. In the theory of structures, the statistical approach of formulating the transition matrix, which is based on historical data of the operating system, is now widespread. Most of them are based on the dependencies of the probability theory obtained by J. R. R. Tolkien. Bogdanoffim and F. Kozin [12-14], where the distribution of discrete states for each year $\mathbf{W}(t)$ is obtained by multiplying the distribution of the previous state by transition matrix $\mathbf{P_0}$:

$$\mathbf{W}(t) = \mathbf{W}(t-1) \times \mathbf{P_0}, \tag{29}$$

where $\mathbf{W}(t-1)$ – a vector of the previous state distribution.

Let's use the simplest of them – estimating the relative number of bridges in each of the states. The elements of the transition probability matrix are determined by formula:

$$p_{ij} = n_{ij} / n_i: \tag{30}$$

where n_{ij} – the number of clicks from state i into state j within a given time period; n_i – the total number of bridges in the state at the beginning of the specified time period.

Let's show the procedure to obtain a transition probabilities matrix using the example of statistical data obtained from the road bridge operation system. Let's look at the historical data of reinforced concrete bridges of all types in operation. The state distribution of all types of bridges is shown in Table 1 below.

Table 1– Distribution of road bridges by operational condition

State	1	2	3	4	5	Total
In absolute terms, units.	112	758	4288	1751	122	7031
In percentage, %	1.6	10.8	61.0	24.9	1.7	100

From Table 1 we obtain a super diagonal elements' vector of the transition probability matrix **P**: $\mathbf{p}_{ij}$, i = 1, 2, 3,…, 4; $j = i + 1$.

$$\mathbf{p}_{ij} = [0.016\ \ 0.108\ \ 0.610\ \ 0.249]^{\mathrm{T}}. \tag{31}$$

The corresponding transition probability matrix will have value:

$$\mathbf{P} = \begin{bmatrix} 0,984 & 0,016 & 0 & 0 & 0 \\ 0 & 0,892 & 0,108 & 0 & 0 \\ 0 & 0 & 0,390 & 0,610 & 0 \\ 0 & 0 & 0 & 0,751 & 0,249 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{32}$$

The transition probability matrix of one year in operation is calculated as $\mathbf{P}^2$:

$$\mathbf{P}^2 = \begin{bmatrix} 0,968 & 0,030 & 0,002 & 0 & 0 \\ 0 & 0,796 & 0,138 & 0,066 & 0 \\ 0 & 0 & 0,152 & 0,696 & 0,152 \\ 0 & 0 & 0 & 0,564 & 0,436 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{32a}$$

The transient probabilities matrix predicted after 5 years in operation will have value:

$$\mathbf{P}^5 = \begin{bmatrix} 0,923 & 0,062 & 0,008 & 0,006 & 0,001 \\ 0 & 0,565 & 0,120 & 0,220 & 0,096 \\ 0 & 0 & 0,009 & 0,388 & 0,603 \\ 0 & 0 & 0 & 0,239 & 0,761 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{32б}$$

Let's introduce a 100-point rating system. (An important note: the technical condition criterion in this model is a rating assessment of the technical condition of the structure), Table 2.

Table 2 – Rating assessment of the structure in operation

State in operation	Evaluation scale, points
State 1. Workable	100 - 79
State 2. Limited workable	80 - 59
State 3. Workable	60 - 39
State 4. Limited workable	40 - 19
State 5. Unworkable	≤ 20

Algorithm forecasting the technical condition of structures. The algorithm for implementing the Markov chain model for predicting the technical condition of the structure as a whole is given in Table 3.

Table 3 – Algorithm for predicting the technical condition of hydraulic structures

Step	Operation
1	Calculating super diagonal elements of the transition probability matrix with using formula (4.30): $p_{ij} = n_{ij} / n_i$ where n_{ij} – the number of clicks from state *i* into state *j* within a given time period; n_i – the total quantity of bridges in the state *and* at the beginning of the specified time period. Diagonal elements are calculated as an addition to 1.
2	The initial state of the system is set by the tape-matrix $\mathbf{p}_0$ by form (27).
3	The system state vector is calculated for time *t* as a product of the transition matrix **P** in degree *t* on the initial state vector of the system by formula (26) $\mathbf{p}_t = \mathbf{p}_0 \text{ X } \mathbf{P}^t$.
4	A defined states vector is introduced **d** in size [*n* X 1] (a safe operation rating vector) by formula (27) $\mathbf{d} = [r_1, r_2, \ldots, r_n]^{\mathrm{T}}$.
5	Technical condition of the structure for any time *t* is defined by dependency (28) $D_{\mathrm{t}} = \mathbf{p}_0 \text{ X } \mathbf{P}^t \text{ X } \mathbf{d}$, where D_{t} – rating assessment of the structure for time *t* – scalar; p_0 – matrix-tape in size [1 X *n*] probabilities in the initial state t_0; $\mathbf{P}^t$ – transition probabilities matrix P in *degree t* in size [*n* X *n*].

The Markov phenomenological model of damage accumulation. The Markov models of random processes described above are universal. The random process described by the model is invariant to the type of modeling object, to the material, and to the operating conditions. As for the failure intensity parameter, it is the subject of a special study in the phenomenological model of damage accumulation, its definition in our model is given below.

The task is to develop a phenomenological probabilistic model of the degradation of a structural element in operation. The element degradation model aims to establish the law of reliability in the time function and, thereby, give an apparatus for predicting its technical condition. The model developed by us has two components: the phenomenological classification tables of discrete states and the reliability function. The following four hypothetical propositions form the theoretical basis of the model.

A. The numerical reliability parameter is taken as a criterion for the technical condition of an element.

B. The element's life cycle is divided into 5 discrete states. Each of the states is described by a selection of quantitative and non-formalized qualitative degradation indicators that characterize the hierarchy of the element failures.

C. The process of element degradation during the life cycle is described by a discrete model of a random Markov process with continuous time.

D. The transition from one discrete state to another is described as a Poisson process with discrete states and continuous time by formula (25).

Discrete states of the element. The system evolution will be described by *the Markov discrete process with continuous time* [5, 6]. Let's formulate the Markov process for models in which wandering through discrete states is carried out only in one direction: from the state with a smaller number to the state with a larger number. At the same time, the transition is possible not only to the neighbouring state but also by "jumping" through neighboring states. In terms of the discrete Markov process, the problem is reduced to the search for unconditional probabilities of finding a system **S** at an arbitrary step *k* in state S_i.

The system of failures, which is a consequence of the wear and tear of the structure element, will be considered a stream of random discrete Markov chain events. The process with "qualitative states" is considered. The role of a random variable is played by the "random discrete state of the system".

During the life cycle of an element in operation let's introduce 5 discrete states that form a tuple $\mathbf{S} = \{S_1, S_2, \ldots, S_5\}$. Discrete states are described by a selection of qualitative and quantitative indicators of no formalized degradation indicators that characterize the hierarchy of element failures

[7, 13]. A generalized description of states that represent damage accumulation as a hierarchy of gradual element failures is given in Table 4.

Table 4 – General characteristics of states

State	State characteristics
S_1	The element meets all the project requirements.
S_2	The element partially does not meet the requirements of the project, but the requirements of either the first or second groups of limit states are not violated.
S_3	The element partially does not meet the project requirements, but the requirements of the first group of limit states are not violated. A partial violation of the requirements of the second group of limit states is possible, if this does not limit the normal functioning of the structure.
S_4	The element has signs of violations of the requirements of the first group of limit states and serious violations of the requirements of the second group of limit states.
S_5	The element has serious violations of the requirements of the first group of limit states and it turns out that it is impossible to prevent them and stop its operation.

Phenomenological description of the process of element degradation of a hydraulic structure. The process of element degradation during the life cycle is described by discrete state classification tables. These tables, depending on the type of material and the design purpose of the element, are compiled on the basis of the experience of expert scientists [5, 7] on supervision, diagnostics, inspection and the testing of structures. In Table 5 an example of describing discrete states of construction structure elements is given.

Table 5 – Operational conditions of pre-stressed reinforced concrete elements

State	Defect or violation	Wear level, %
1	Single chipping of small rebar sizes in concrete Single sinks in the concrete of small sizes without exposing the reinforcement Single hair cracks without rust marks with opening up to 0.2 mm Hydrogen index *ph*=11	0-1
2	Local temperature-shrinkage cracks with opening up to 0.1-0.2 mm Local chipping of concrete without exposing rebar Local sinks without exposing fittings Local smudges without exposing the reinforcement Hydrogen index *ph*=10	2-4
3	Numerical chips in the stretched area of the structure Numerous sinks in stretched concrete Traces of leaching on the element surface Hydrogen index *ph*=9 Single opening of force cracks in inclined sections or along the reinforcement	5-14
4	Cracks in the stretched concrete with an opening of more than 0.2 mm Inclined force cracks in support zones Temperature cracks in support zones Traces of concrete leaching on the element surface Hydrogen index *ph*=8	15-33
5	Longitudinal cracks in compressed concrete along pre-stressed reinforcement with peeling of the concrete protective layer Traces of rust near cracks Uneven deflection of bent elements Hydrogen index *ph*=7	≥ 34

Reliability function. The reliability function describes the process of element degradation during the life cycle, that is, it establishes a relationship between reliability and the service life of the element. It is postulated that degradation speed is described by one parameter λ – an indicator of the failure rate. This indicator is assumed constant, independent of time $\lambda = \lambda\,(t)$.

For the reliability function according to hypothesis **D,** the Poisson distribution law is adopted. When $k = 5$ the function has the form:

$$P(t) = 1 - 0{,}008333 \cdot \left(\lambda(t)t\right)^5 e^{-\lambda(t)t}. \tag{33}$$

where P_t – the probability that the element will enter state k within the time $t < T_k$.

Thus, at a given failure rate λ, dependence (33) establishes a relationship between the reliability of the element P_t in i- state and time t, passed from the start of the operation to state $i=2,...5$.

Transition probabilities matrix. Model A. The model is represented by the process, whose graph is shown in Fig. 1. This is a discrete process with continuous time. The system can sequentially move from one neighbouring state to another with a larger number, or stay in any of them. State 5 is absorbing. This means that the system does not have the exit of state 5.

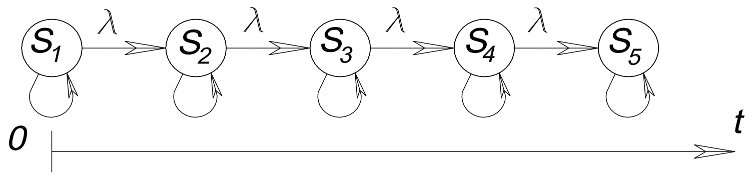

Fig. 1. Model A. Process graph

Let's define matrix **P**(i, t) and matrix **E** by the Kolmogorov–Chapman equations (17). To simplify writing matrix elements, their shape will be changed slightly. In the future, the argument t with the transition probability $p_i(t)$, i = 1, 2, 3, 4 and transition intensities $\lambda(t)$ and argument (i, t) at matrix **P**(i, t) will not be written.

In model A, let's set the transition intensities independent of the step and time:

$$\lambda_{ij}(t) = \lambda(t) = \lambda\,. \tag{34}$$

Matrix **E** will look like this:

$$\mathbf{E}^T = \left[\lambda\ \lambda\ \lambda\ \lambda\ \right], \tag{35}$$

and equation (20) in this case is simplified and written as follows:

$$\frac{d\mathbf{P}}{dt} = \lambda\mathbf{P} \tag{36}$$

According to the process graph of model A, the system of equations (36) will be:

$$\frac{d\mathbf{P}}{dt} = \lambda \cdot \begin{bmatrix} -p_1 & 0 \\ -p_2 & p_1 \\ -p_3 & p_2 \\ -p_4 & p_3 \end{bmatrix}, \tag{37}$$

Integrating the system of equations (37) under conditions (21, 22) gives the values of the transition matrix elements:

$$\mathbf{P}_0 = \begin{bmatrix} 0{,}9900 & 0{,}0100 & 0 & 0 & 0 \\ 0 & 0{,}9802 & 0{,}0198 & 0 & 0 \\ 0 & 0 & 0{,}9704 & 0{,}0296 & 0 \\ 0 & 0 & 0 & 0{,}9608 & 0{,}0392 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{38}$$

The initial unconditional probability is accepted p_k = 0.9998 (β=3.8. Here β is the safety characteristic, and the numerical parameter is related to reliability by the relation: Pt = Φ-β), where Φ is the standard function of the normal distribution), which corresponds to the minimum standard value of the design reliability in state 1. By formula (21), an unconditional probabilities vector is obtained in state j:

$$\mathbf{P_j} = \begin{bmatrix} 0{,}9998 & 0{,}9899 & 0{,}9703 & 0{,}9416 & 0{,}9047 \end{bmatrix}^T. \qquad (39)$$

According to the known unconditional probabilities vector of the system in state j = 1, 2,..., 5 degradation curves are defined, i.e. a family of implementations of a stochastic process, each of which, at a given value of the failure intensity parameter, λ gives *transition time forecast* from state j into state j + 1 [8, 9].

Model B. Model A describes the stochastic process of gradual accumulation of damage. In reality, the ageing process of a structure element consists not only of gradual failures but also of sudden ones. This is exactly what model B is. In it the process of damage accumulation contains sudden "jumps" over one state, as shown in the process graph Fig. 2.

It is also a discrete process with continuous time, with evenly distributed time intervals between the states. The system can remain in any of them, sequentially move from one neighbouring state S_i to another one with a larger number S_{i+1}, or jump over a nearby state S_{i+1} to the next one S_{i+2}. State 5 – absorbing.

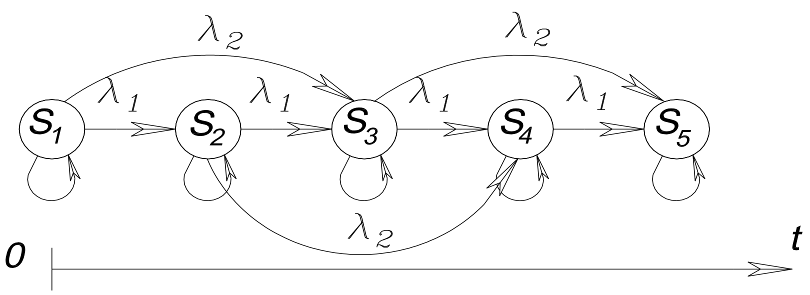

Fig. 2. Model B. Process graph

According to the process graph of model B (fig. 2) the system of equations (17) will be:

$$\frac{d\mathbf{P}}{dt} = \begin{bmatrix} -\varphi p_1 & 0 & 0 \\ -\varphi p_2 & \lambda_1 p_1 & 0 \\ -\varphi p_3 & \lambda_1 p_2 & \lambda_2 p_1 \\ -\lambda_1 p_4 & \lambda_1 p_3 & \lambda_2 p_2 \end{bmatrix}, \qquad (40)$$

here λ_1 transition intensity from state S_i in state S_{i+1}; λ_2 – transition intensity from state S_i in state S_{i+2}; $\varphi = \lambda_1 + \lambda_2$.

The transition intensity to the neighbouring state is assumed λ_1 and the "jump" transition intensity $\lambda_2 = 0{,}05 \cdot \lambda_1$. The numerical solution of the system of differential equations (40) by the Runge-Kutt method gives the values of conditional transition probabilities from which the transition matrix is formed:

$$\mathbf{P_0} = \begin{bmatrix} 0{,}9851 & 0{,}0049 & 0{,}0100 & 0 & 0 \\ 0 & 0{,}9704 & 0{,}0097 & 0{,}0199 & 0 \\ 0 & 0 & 0{,}9560 & 0{,}044 & 0 \\ 0 & 0 & 0 & 0{,}9418 & 0{,}0582 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (41)$$

The initial unconditional probability is assumed, as for model A $p_k = 0.9998$ ($\beta = 3.8$) and by the known values of conditional transition probabilities $\mathbf{P_0}$ and by formula (24) an unconditional probabilities vector is obtained in state j of a stochastic system of model type B:

$$\mathbf{P_j} = [0{,}9998 \quad 0{,}9849 \quad 0{,}9558 \quad 0{,}9138 \quad 0{,}8605]^T . \tag{42}$$

As expected, the probability in states 2-5 is lower compared to model A, which can be interpreted as an implementation in the process of sudden failures.

Determining the failure rate parameter. Determination of the failure intensity parameter is the dominant feature of the Markov phenomenological model of accumulation of damage to construction structure elements. As seen from dependency (36), the only parameter of lifecycle management is the failure rate λ. In the model under consideration, the parameter λ is determined by solving equation (33) under the initial conditions for an individual element obtained from the survey results. The procedure for determining the parameter λ was first proposed in 1999. It is in a specific definition of the initial conditions for equation (36) with respect to an unknown parameter λ:

– reliability $P_{t,i}$ is related to a specific task i-th technical condition. This value becomes known as soon as the discrete state of the element is classified according to the survey data, and t_i – the time elapsed from the start of operation of the element to state i. Time t_i is known from the technical documentation of the bridge. A graphical interpretation of the parameter determination procedure is given in Fig. 3.

It is obvious that the proposed method for determining the parameter value λ of a structural element that controls the "*nowadays"*, that is, with *i-th* discrete state, a life cycle model, provides complete information about the load history in "*the past*", and not only that. The failure rate parameter defined by this procedure λ *contains a lot of other information about the operation of the facility*, related to the characteristics of the environment, the level of loading effects, the quality of construction, design features, etc.

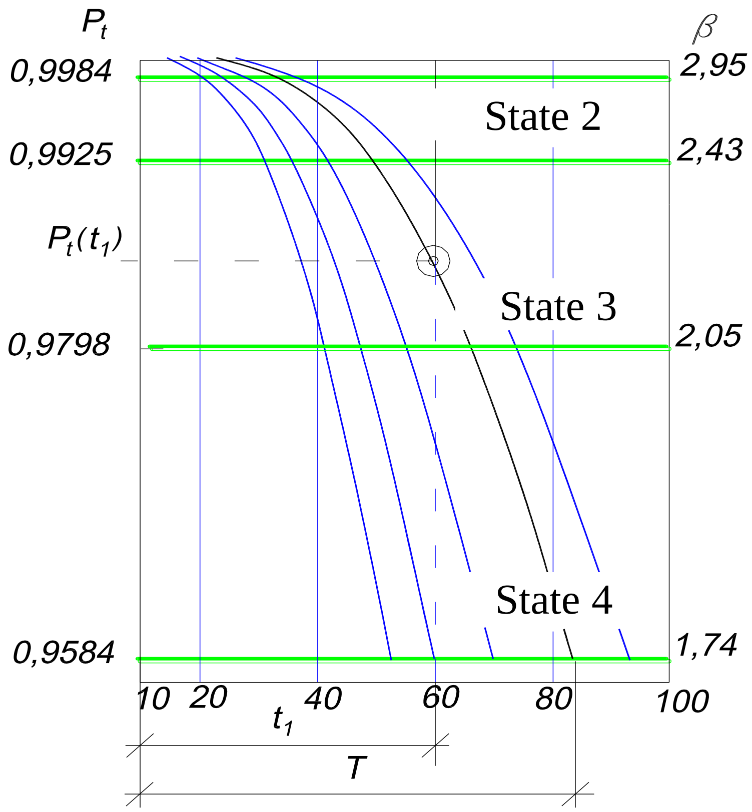

Fig. 3. Graphical interpretation of the failure intensity parameter definition λ

Predicting the remaining resource of the element. The forecast of the remaining resource is determined, again, by solving the degradation equation (36). The initial data for solving the equation is now the reliability of element $P_{t,5}$ – the limit value of reliability in the 5th operational state and the parameter of the element failure rate determined in the previous step from equation (36) λ.

The time determined under these initial conditions is T_5 – that is, the time that passes from the current state of the element to the fifth operational state and is the remaining resource.

An example of the model implementation is the new regulatory documents for the system of road bridges' operation in Ukraine [5, 6].

Analysis of model. A. As seen from the above formulation, the model is theoretically strictly justified. However, the fact that the model is phenomenological requires a deep insight into the physical essence of the described process, because it is necessary not only to adequately describe each of the discrete states but also to establish changes in parameters correctly within one discrete state, while the simulated process is continuous.

Б. The decision on the number of discrete states representing the life cycle of an element is quite subjective. It is clear that the more discrete states there are, the more accurately the continuous process of damage accumulation is described. On the other hand, describing a large number of discrete states requires a significant expansion of the database of reliable full-scale data. The model developer decides where the reasonable satisfaction of these conflicting requirements is.

В. An important theoretical side of the model is the graph of the element degradation process. The graph of the model, which depends on the number of discrete states and the connections between them, will always be the subject of special attention on the part of the researcher, and will always reflect his subjective idea of the essence and regularities of the process.

Г. Another fundamentally important aspect of the model is the question of the developer's definition of the failure intensity function λ(t), which is generally a random function of time. However, there is no standard procedure here, and the researcher has to look for special techniques for determining this basic parameter of model control.

The economic effect of forecasting the resource of hydraulic structures. Assessment of the possible economic effect of forecasting the resource of construction structures is performed according to criterion $Z_{зр}$, which takes into account the operating costs of managing the object's state. The minimum goal function is obtained with *using the penalty method*:

$$Z_{зр} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{C_{i,t}(S_{i,t})}{BI_{i,t}\cdot(1+r)^t}\cdot\left[1+\max\left(\frac{\beta_{\min}-\beta_{i,t}}{\beta_{\min}},0\right)\right]\right) => \min, \tag{43}$$

where $C_{t,i}$ – operating expenses for managing the condition of the construction structure i per year t, UAH; $S_{i,t}$ – operational condition; β_{min} – minimum allowable value of the safety characteristic; $\beta_{i,t}$ – a value of the safety characteristic of the *i-th* construction structure per year t; r – discount rate adjusted for the inflation rate, $BI_{i,t}$ – importance factor (priority) i-th element at the beginning of the year t.

The time-dependent priority of a construction structure is determined by formula:

$$BI_k(t) = \sum_{i=1}^{N_{Ft}} w_i \cdot f_{ik}(t) + \sum_{i=1}^{N_F} w_i \cdot f_{ik}, \tag{44}$$

where N_{Ft} – the number of factors of influence that depend on time t, in years; N_F – the number of factors of influence that do not depend on time; w_i – normalized impact factor *i-th* factor, (established by experts using the Saati method, $\sum_{i=1}^{N_F} w_i = 1$); f_i – dimensionless value of importance *i*-the structure element determined by an expert.

Thus, criterion $Z_{зр}$ – is the number of operating expenses adjusted for the penalty ratio. If the safety characteristic β falls below the specified minimum allowable value of the function value $Z_{зр}$ will increase, which serves as a barrier to an unjustified decrease in the permissible value of $\beta_{\min}$

security features. Through fines, the achieved condition of the facility is taken into account, and better condition means minimizing operating costs and maximizing external positive effects.

The minimum allowable level β_{min} is set based on an economic justification, but it cannot be less than the maximum permissible limit for the safety requirements for the operation of a construction structure element, for example, less than 1.74.

Conclusions. Assessment of the possible economic effect of forecasting the resource of hydraulic structures is performed according to Z criterion$_{зр}$. It represents the number of operating expenses adjusted for the penalty ratio. When the safety characteristic β falls below the specified minimum allowable value, the value of the function $Z_{зр}$ will increase, which serves as a barrier for unjustified reduction of the allowable value β_{min} of the safety characteristic. Through fines, the achieved condition of the facility is taken into account, and better condition means minimizing operating costs and maximizing external positive effects. The minimum allowable level β_{min} is set based on the economic justification, but it cannot be less than the maximum permissible limit for the safety requirements for the operation of hydraulic structures.

The reliability function describes the process of degradation of hydraulic structures during the life cycle, that is, the relationship between reliability and the operating time of the element is established. It is postulated that the degradation rate is described by one parameter – an indicator of the failure rate. This indicator is assumed constant, independent of time.

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ТЕОРЕТИЧНІ ЗАСАДИ ПРОГНОЗУВАННЯ РЕСУРСУ КОНСТРУКЦІЙ ГІДРОТЕХНІЧНИХ СПОРУД

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Анотація. Встановлено, що в термінах дискретного марковського процесу задача зводиться до пошуку безумовних ймовірностей перебування системи S на довільному кроці k в стані S_i, тобто отримання матриці перехідних ймовірностей. В такому формулюванні модель служить для: оцінки технічного стану елемента; оцінки рівня безпеки експлуатації елементів конструкції; ранжирування елементів за потребою ремонтів, реконструкції або заміни; в стратегічному плануванні видатків на ремонт або реконструкцію за умов обмеженого фінансування та прогнозу залишкового ресурсу елементів.

Встановлено, що теоретичним базисом дослідження, що має за мету прогнозування ресурсу гідротехнічних споруд у процесі експлуатації, є марковська теорія випадкових процесів. Для математичного опису процесу деградації елементів найбільш вдалим є математичний апарат випадкових марковських процесів.

Визначення параметра інтенсивності відмов є домінантою марковської феноменологічної моделі накопичення пошкоджень елементів гідротехнічних споруд. Єдиним параметром управління життєвим циклом є інтенсивність відмов λ. В моделі, що розглядається, параметр λ визначається за початкових умов для окремого елементу, отриманих за результатами обстеження.

За рахунок того, що параметр λ визначається для окремого елемента і має уточнюватися кожного разу після чергового обстеження, точність моделі підвищиться. Модель, що пропонується, є інтегральною. Вона не містить явного теоретичного апарату чуйного до матеріалу елементу, його статичної схеми, технології спорудження, екологічних умов та такого іншого. З іншого боку, всі названі фактори і багато інших, другорядних, беруться до уваги в моделі в момент, коли за допомогою класифікаційних таблиць, що містять фізичні і механічні ознаки деградації, визначається стан елемента.

В теорії споруд поширеним є статистичний підхід формулювання матриці переходів, в основі якого лежать історичні дані системи експлуатації споруди. Вважається, що матриця переходів розбудована за даними системи експлуатації є більш реалістичною основою для прогнозу процесів деградації споруд. Практичному застосуванню матриці переходів розбудованої за статистичними даними присвячена велика кількість зарубіжних досліджень в яких розглядаються особливості матриць переходів пов'язані з системою експлуатації мостів різних країн. В такій постановці кожен елемент матриці перехідних ймовірностей P є ймовірність того, що система яка була в стані i перейде в стан j за один крок (тобто за один рік). При цьому вважається що відсутні експлуатаційні втручання, тому піддіагональні елементи є нульовими. Як і раніше сума елементів одної строки дорівнює 1 і елемент $p_{jj} = 1$ тому як стан *j* є поглинаючим.

Для алгоритму реалізації моделі марковського ланцюга для прогнозування технічного стану гідротехнічних споруд в цілому вихідними даними є: статистичні дані розподілу споруд по станам на час прогнозу, рейтингова оцінка споруди обчислена експертом згідно шкали та час прогнозу в роках.

Встановлено, що деградаційні властивості конструкцій споруд описуються двома параметрами: критерієм деградації та інтенсивністю відмов. Критерієм деградації може бути прийнятий будь-який фактор напружено-деформованого стану: надійність, внутрішні зусилля, деформації. Критерієм деградації може виступати довільна рейтингова оцінка. В нашому випадку за критерій деградації приймається надійність елемента, як найбільш загальний фактор напружено-деформованого стану.

Ключові слова: гідротехнічні споруди, деградація конструкцій споруд, Марковська модель, термін служби.

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