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DEPENDENCE OF TENSILE FORCE OF THE ELASTIC LIMIT STATE OF ROPES FROM THE BENDING PARAMETER ON THE DRUM

[1]**Vovk P.E.,** Graduate student,
vovk.pavel.1995@gmail.com, ORCID: 0000-0001-6156-1686
[1]**Chaiun I.M.**, DSc., Professor,
i.m.c@ukr.net, ORCID: 0000-0003-0867-8791
[1]*Odessa National Polytechnic University*
Shevchenko avenue, 1, Odessa, 65044, Ukraine

Abstract. Based on the method previously developed by the authors for the analytical determination of the ultimate elastic state of the ropes, the dependence of the $P(\overline{D})$ tensile force with the winding of the rope on the drum on the parameter $\overline{D} = D_d/d_r$ (the ratio of the diameters of the drum and the rope) in the interval $\overline{D} = 10 - 120$. Dependence $P(\overline{D})$ was determined for two loading schemes of rope: stretching of the rope with winding on the drum with a freely suspended load and stretching of the rope with winding on the drum with the load in the guides. Based on the developed method, the dependence of $P(\overline{D})$ was performed for 16 kantais of different designs. The tensile strength is presented in the relative form $\overline{P}(\overline{D}) = P(\overline{D})/P_t$ (P_t total breaking strength of the rope wires). The effort $\overline{P}(\overline{D})$ significantly depends on the construction of the rope and the bending parameter $\overline{D}$. In the section $40 \leq \overline{D} \leq 120$ the force $\overline{P}(\overline{D})$ increases monotonically and practically linearly, reaching the value corresponding to the calculation scheme of stretching a straight rope. For different structures with the parameter $\overline{D} = 40$ change in force $\Delta\overline{D} = 0.723 - 0.578 = 0.145$ with $\overline{D} = 120 \cdot \Delta\overline{P} = 0.765 - 0.677 = 0.08$. In the section $40 \geq \overline{D} \geq 20$ the dependence $\overline{P}(\overline{D})$ is not linear, at $\overline{D} = 40 \cdot \Delta\overline{P} = 0.663 - 0.418 = 0.245$. The section $20 \geq \overline{D} \geq 10$ is characterized by a sharp change in force $\overline{P}(\overline{D})$, with the bending parameter $\overline{D} = 10 \cdot \Delta\overline{P} = 0.416$. For most rope designs at $\overline{D} < 10$ the forces $\overline{P}(\overline{D})$ are close to zero values. When stretching with a freely suspended load, the forces $\overline{P}(\overline{D})$ are 1.6–1.7 times lower than when stretching in guides. For twisting ropes (one-way winding), the ratio is 2.5–3.4 times. In the normative methods of calculations of lifting ropes, the characteristics P_t or $P_a = 0.83P_t$ are used, which do not take into account the peculiarities of the deformation and construction of the ropes. We believe that the given information is appropriate in solving the issue of building a methodology for calculating the static strength of lifting ropes based on the characteristics of their ultimate elastic state, which will ensure stable optimality of the use of ropes, will allow you to rationally choose the type of rope construction and its dimensions.

Keywords: rope, strength characteristics, calculation scheme, bending parameter when winding on a drum, calculation method for static strength.

Introduction. The strength calculations of lifting ropes use characteristics corresponding to extremely simplified calculation schemes [1–4]. One of them, the most simplified, is the total breaking force:

$$P_t = F_t \sigma_b, \tag{1}$$

where F_t – total area of all wires of the rope;

σ_b – wire strength limit (marked rope group).

The second is the aggregate breaking force of the rope as a whole:

$$P_a = kP_t, \tag{2}$$

where k is a coefficient, which in various sources [1–4] has close, slightly different numerical values. So, in [1, 2, 4] $k = 0.83$ in [3] $k = 0.75 - 0.90$ (a smaller value refers to three-layer double-wound ropes, more to single-wound ropes).

In the European standard EN 12385-2, 10 indicators of strength characteristics are established for ropes. Two of them fully correspond to the specified characteristics P_t and P_a, the others are based on these two characteristics and are also not connected by real load schemes of the lifting ropes. According to the content, it seems that the characteristics of EN 12385-2 are intended for comprehensive control of rope manufacturing technology.

Such inaccuracy (simplification) of the calculation schemes of the ropes is compensated by an increase in the level of normative safety margins, the interval of which is $[m] = 3 - 13$, according to [1, 2, 4]. We consider the statement in [4, p. 96] that high values of [m] "reflect greater responsibility of steel ropes". Of course, the responsibility, but no less important is the factor of extreme simplicity of the calculation scheme of the ropes. Under such conditions, the calculation of ropes cannot be stably optimal.

Research analysis. In work [1], the calculation of the strength of mine lifting ropes is reduced to a comparison of the normative margin of strength [m] with the estimated:

$$m = \frac{P_t}{S} = \frac{qg\sigma_b}{(Q_k + qL)\gamma}, \quad (3)$$

where S estimated static load;

q – linear rope mass, kg/m;

g – acceleration of free fall, m/s^2;

$\gamma = q/F_t 10^6$ – fictitious (reduced) density;

$Q_k = Q_b + Q_c$ – final mass, which includes the mass of the cargo and the mass of all suspended elements;

L – lift height.

Formula (3) does not take into account the bending of the rope on the drum and blocks.

In Rules [2], the calculation is reduced to checking the selected rope according to the formula:

$$P_a \geq z_p S, \quad (4)$$

where $P_a -$ the second is the aggregate breaking force of the rope as a whole; S – the greatest tension of the rope branch; z_p – minimum utilization factor (safety factor).

In the Rules [2], the influence of rope bending is somewhat taken into account by assigning the minimum diameters of drums, blocks and leveling blocks according to the formulas:

$$D_1 \geq b_1 d;\ D_2 \geq b_2 d;\ D_3 \geq b_3 d, \quad (5)$$

where $D_1; D_2; D_3$ – diameters of drums and blocks;

$b_1; b_2; b_3$ – assignment coefficients for drum and block diameters; d – rope diameter.

Table 1, which combines two tables from the Rules [2], shows the correspondence between the normative safety margins and the parameters $\overline{D} = D_d/d_r$. But this correspondence is not sufficient, because it does not take into account the construction of the ropes, the deformed-stressed state and does not give the connection of the parameter $\overline{D}$ with their strong characteristics.

Table 1 – Minimum safety margins and rope bending parameters $\overline{D}$

Mechanism class according to ISO 4301/1	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
Coefficients of margin of strength of the rope z_p	3.15	3.35	3.55	4	4.5	5.6	7.10	9
$b_1(\overline{D} = D_d/d_r)$	11.2	12.5	14	16	18	20	22.5	25.0
$b_2(\overline{D} = D_d/d_r)$	12.5	14.0	16.0	18.0	20	22.4	25.0	28.0
$b_3(\overline{D} = D_d/d_r)$	11.2	12.5	12.5	14	14	16	18	18

In article [3], where the current state of the ropes in crane installations is analyzed, their calculation for static strength is reduced to a condition of the form (4), but with a different interpretation of the component z_p (he is named "minimum design factor", although in terms of the terms (4) – this is a normative safety factor, it is also taken depending on the class of the mechanism and the type of rope).

In article [4] the calculation is reduced to the appointment of the diameter of the rope according to the condition:

$$[F_0] \geq F_0 = F_{\max} z_p,$$

where $F_0 -$ the second is the aggregate breaking force of the rope as a whole;

$F_{max} -$ the calculated maximum force in the rope; $z_p -$ is the safety coefficient, which is also based on the formula (4).

This means that in the methods considered in the works [3, 4], the strength characteristics of the ropes are taken, which also do not take into account the bending deformation when the rope is wound onto the drum.

In work [5], on the basis of formula (4), it is proposed to determine the value of the breaking force, according to which the rope is selected, taking into account operational features (diameters of the drum, blocks, method of securing the rope, intensity of work, etc.). But here the deformed-stressed state of the ropes is not taken into account too.

In Germany [6], in order to increase the service life of the ropes, it is recommended to assign the parameter $\overline{D}$ depending on the lifting speed: at a speed greater than 4 m/s, the parameter $\overline{D} \geq 100$ at a speed lower than 4 m/s, the parameter $\overline{D} \geq 60$ These ratios apply to ropes of all structures except closed, for which $\overline{D} \geq 120$ is recommend.

In the USA, according to the Federal Law "Occupational Safety and Safety", the following standards are established for mining installations designed for the transportation of people: $\overline{D} \geq 60$ when the diameter of the rope is less than 25.4 mm; $\overline{D} \geq 80$ when the diameter of the rope is more than 25.4 mm; $\overline{D} \geq 100$ for closed ropes.

Similar offers are also indicated by rope manufacturers. So, the English company "Bridon" recommends a parameter $\overline{D}$ in the range of 100-120 for closed ropes of its production. In the past, the Khartsyz steel wire and rope plant for closed lifting ropes with a diameter of 20-38 mm GOST 10506 recommended the following ratios: for ropes with a diameter of $d \geq 27$ mm parameter $\overline{D} \geq 90$ for ropes with a diameter of $d \geq 30$ mm parameter $\overline{D} \geq 120$.

It can be seen from the given analysis: consideration of the bending of lifting ropes is somewhat assumed, but without connection with their design, with the external and internal deformed-tension state of the ropes, and there is no connection of the parameter $\bar{D}$ with the corresponding strength characteristics of the ropes.

Aim of the study. The aim of the work is an analytical study of the dependence of the tensile force of the ropes (those their strength characteristics) in the limiting elastic state of the bending parameter associated with winding on the drum under conditions of loading with a load in the guides and with free suspension. Ultimately, the aim is subordinated to the proposal to use such characteristics in the methods for calculating ropes for static strength. This will provide a significant increase in the accuracy of the design scheme of the ropes in comparison with the existing regulated methods, which are based on the strength characteristics that do not correspond to the loading schemes of the ropes.

Research materials and methods. The finite element method was used to study the strained state of the rope. Two schemes of rope loading are considered: stretching of the rope with winding on the drum with a freely suspended load and stretching of the rope with winding on the drum with the load in the guides.

Research results. In this work, on the basis of external and internal deformed states, it is proposed to study the dependence of the tensile force $P_e(\overline{D})$ of the limit elastic state of the rope on the bending parameter $\overline{D} = D_d/d_r$ when winding on a drum. Fig. 1 shows the schemes of the loaded and deformed state of the lifting rope, which are characterized by the bending parameter $\overline{D}$ and internal force factors (IFF) in sections: N – longitudinal force; M, L, Γ torque and bending moments. The solution of the given task is based on the theoretical foundations of works [6, 7]. Let's consider their relationship with the final formula of dependence $P_e(\overline{D})$ on the basis of the deformed state corresponding to the diagrams (Fig. 1) of the rope load.

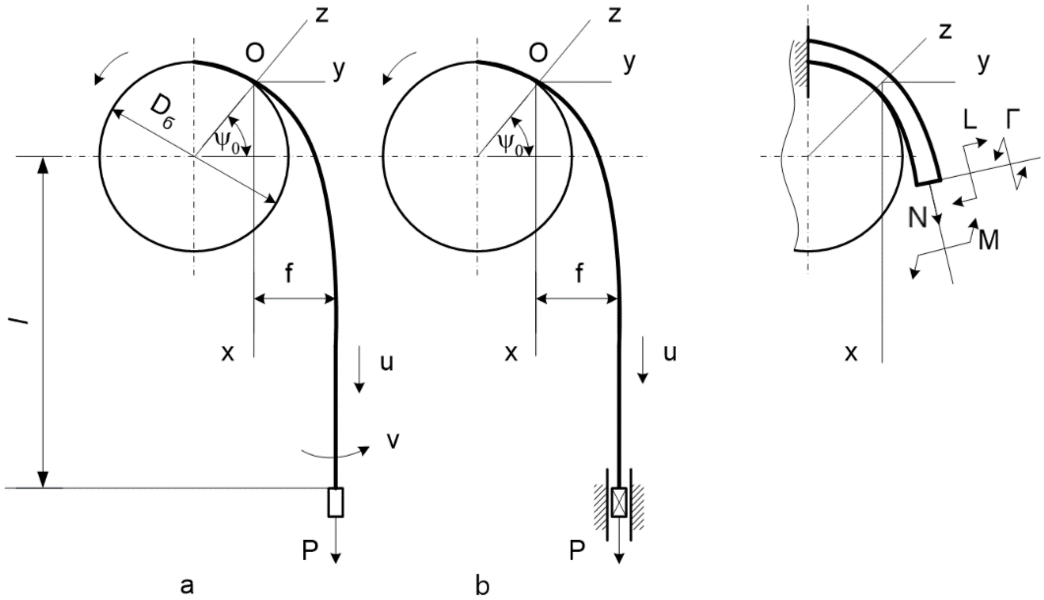

Fig. 1. Rope load schemes:
a – the load is freely suspended; b – the load is in the guides

The external deformed state of the rope is expressed by the vector of deformations:

$$|DK| = |\varepsilon\theta\chi\xi| = |G|^{-1}|F|. \tag{6}$$

Here $\varepsilon, \theta, \chi, \xi$ – according to stretching, torsion, and bending deformations (in the elastic state, the components of the vector $|F|$ IFF are proportional to them;

$|G|$ – global stiffness matrix of the rope section:

$$|G| = \begin{vmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{11} & G_{43} & G_{44} \end{vmatrix} = \sum_1^S sc\ \alpha_i\ sc\beta_i |K|_i \begin{vmatrix} \Phi_{p_i} & & & 0 \\ & \Phi_{t_i} & & \\ & & \Phi_{u_i} & \\ 0 & & & \Phi_{u_i} \end{vmatrix} |K|_i^T, \tag{7}$$

where $G_{11}, G_{22}, G_{33}, G_{44}$ – the main stiffnesses of the rope section (longitudinal, torsional, bending);

$G_{12} = G_{21}$; $G_{13} = G_{31}$ etc. – severity of impact;

$\Phi_{p_i} = \mathrm{E}\pi\delta^2/4$; $\Phi_{t_i} = \mathrm{E}\pi\delta^4/80$; $\Phi_{u_i} = \mathrm{E}\pi\delta^4/64$ – stiffness of the cross section of the wires (longitudinal, torsional, bending; E – modulus of elasticity; δ – diameter of the wires).

Matrix $|K|_i$ specific deformations of the *i*-th wire:

$$|K|_i = \begin{vmatrix} K_{e\varepsilon} & K_{t\varepsilon} & K_{b\varepsilon} & K_{n\varepsilon} \\ K_{e\theta} & K_{t\theta} & K_{b\theta} & K_{n\theta} \\ K_{e\chi} & K_{t\chi} & K_{b\chi} & K_{n\chi} \\ K_{e\xi} & K_{t\xi} & K_{b\xi} & K_{n\xi} \end{vmatrix} = |KT|_i \cdot |KF|_i = \tag{8}$$

$$|K|_i = \begin{vmatrix} \overline{K}_{E\varepsilon} & \overline{K}_{T\varepsilon} & \overline{K}_{B\varepsilon} & 0 \\ \overline{K}_{E\theta} & \overline{K}_{T\theta} & \overline{K}_{B\theta} & 0 \\ \overline{K}_{E\chi} & \overline{K}_{T\chi} & \overline{K}_{B\chi} & \overline{K}_{N\chi} \\ \overline{K}_{E\xi} & \overline{K}_{T\xi} & \overline{K}_{B\xi} & \overline{K}_{N\xi} \end{vmatrix} \cdot \begin{vmatrix} K_{eE} & K_{tE} & K_{bE} & 0 \\ K_{eT} & K_{tT} & K_{bT} & 0 \\ K_{eB} & K_{tB} & K_{bB} & K_{nB} \\ K_{eN} & K_{tN} & K_{bN} & K_{nN} \end{vmatrix};$$

$|KT|_i$ – matrix of specific deformations of the strands in the rope, which includes the i-th wire; $|KF|_i$ – matrix of specific deformations of the i-th wire in the strand.

In the elements of the matrix $|K|_i$, the first index indicates the deformation of the wire in the rope (e – longitudinal; t – torsion; b and n – bending), and the second – the deformation of the rope, from which this deformation of the wire is obtained.

In the elements of the matrix $|KT|_i$, the first index indicates the deformation of the strand in the rope, which includes the i-th wire (E – longitudinal; T – torsional; B and N – bending), and the second index indicates the deformation of the rope, from which this deformation of the strand occurs.

In the elements of the matrix $|KF|_i$, the first index indicates the deformation of the wire in the strand, and the second – the deformation of the strand, with which this deformation of the wire is associated (for a single winding rope, the first index is the deformation of the wire, and the second – the deformation of the rope, for example, $K_{e\chi} = \nu cos^2\alpha\ cos\varphi$).

The elements of the matrix $|KF|_i$ and $|KT|_i$ are obtained on the basis of geometric equations of deformations, respectively, of wires in a single-twist rope (strand) and strands in a double-twist rope [8], taking into account transverse narrowing and friction [6].

Vector $|F|$ of internal power factors in formula (6):

$$|F| = |NML\Gamma|^T, \quad (9)$$

here $N = Pcos\psi$ longitudinal force, $N = Pcos\psi - M = M_H cos\psi$ – torque moment, $L = Pfe^{-kx}$; $\Gamma = M_H sin\psi$ – bending moments;

where $k = (P/G_{33})^{0.5}$;

$$f = G_{33}/PR; \quad (10)$$

x and ψ – linear and angular coordinates of the cross-section of the rope:

$$\psi = (G_{33}/PR^2)^{0.5}e^{-kx}; \quad (11)$$

$M_H = -PA_{12}/A_{22}$ guide reaction (A_{12} and A_{22} algebraic additions of the stiffness matrix G). Bending curvature in the plane of winding on the drum (curvature of the bent axis of the rope):

$$\chi = R^{-1}e^{-kx}. \quad (12)$$

The maximum values of the curvatures and the angle of rotation of the rope section at point O (Fig. 1):

$$\chi_O = \left|2(D_d + d_r)^{-1} = 2\left(d_r(\overline{D} + 1)\right)^{-1}\right|;\ \psi_O = (G_{33}/PR^2)^{0.5}. \quad (13)$$

The internal deformed state of the rope is expressed by a matrix formula:

$$|D\Pi| = |DK||K|, \quad (14)$$

where $|D\Pi|$ – block matrix $1 \times s$ (s – number of wires in the rope).

Matrix components (14):

$$|D\Pi_i| = |e\ t\ b\ n|, i = 1,2, \dots, s, \quad (15)$$

where e, t, b, n – strains of stretching, torsion, bending of wires.

The calculation formula $P_e(\overline{D})$ tensile strength of the ultimate elastic state of ropes for 2 load schemes (Fig. 1) (note that the same formula looks the same for 2 straight rope load schemes) is obtained on the basis of the proportionality between the longitudinal deformation e (15) of the wires in the rope and the elastic limit ε_{lim} of the corresponding wire:

$$P_e(\overline{D}) = \varepsilon_{lim}/max\overline{e}_i;\ i = 1,2, \dots, s, \quad (16)$$

where ε_{lim} – deformation of the elastic limit of the cable wire according to the diagram $\sigma - \varepsilon$ tensile testing of its samples;

$max\overline{e}_i$ – the largest value among all s wires of the rope of the specific tensile strain at the final load $\overline{P} = 1$.

$$max\overline{e}_i = \overline{\varepsilon}K_{e\varepsilon_i} + \overline{\theta}K_{e\theta_i} + \overline{\chi}K_{e\chi_i} + \overline{\xi}K_{e\xi_i}, \quad (17)$$

$\overline{\varepsilon}, \overline{\theta}, \overline{\chi}, \overline{\xi}$ – specific deformations of stretching, twisting, and bending of the rope from $\overline{P} = 1$ for the load schemes (Fig. 1), which are obtained on the basis of the matrix formula (6) using all subsequent dependencies (6) – (13):

$$|DK| = \begin{vmatrix} \varepsilon \\ \theta \\ \chi \\ \xi \end{vmatrix} = \frac{1}{|D|} \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix} \cdot \begin{vmatrix} N \\ M \\ L \\ \Gamma \end{vmatrix}, \quad (18)$$

where the determinant of the stiffness matrix of the rope section:

$$|D| = G_{11}A_{11} + G_{12}A_{12} + G_{13}A_{13} + G_{14}A_{14};$$

algebraic additions:

$$A_{11} = G_{22}(G_{33}G_{44} - G_{34}^2) - G_{23}(G_{23}G_{44} - G_{34}G_{24}) + G_{24}(G_{23}G_{34} - G_{33}G_{24})$$

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$$A_{44} = G_{11}(G_{22}G_{33} - G_{23}^2) - G_{12}(G_{12}G_{33} - G_{23}G_{13}) + G_{13}(G_{12}G_{23} - G_{22}G_{13}).$$

On the basis of (18), the deformations of the rope at a unit load are determined $\overline{P} = 1$.

In the case of stretching when the load is freely suspended (Fig. 1, a):

$$\overline{\varepsilon} = (A_{11}cos\psi + A_{13}fe^{-kx})/|D|;\ \overline{\theta} = (A_{12}cos\psi + A_{23}fe^{-kx})/|D|; \quad (19)$$

$$\overline{\chi} = (A_{13}cos\psi + A_{33}fe^{-kx})/|D|;\ \overline{\xi} = (A_{14}cos\psi + A_{34}fe^{-kx})/|D|. \quad (20)$$

In the case of stretching in the guides (Fig. 1, b):

$$\overline{\varepsilon} = (A_{11}cos\psi - A_{12}(A_{12}cos\psi + A_{14}sin\psi)/A_{22} + A_{13}fe^{-kx})/|D|; \quad (21)$$

$$\overline{\theta} = (A_{23}fe^{-kx} - A_{24}\frac{A_{12}}{A_{22}}sin\psi)/|D|; \quad (22)$$

$$\overline{\chi} = (A_{13} - A_{12}(A_{23}cos\psi + A_{34}sin\psi)/A_{22} + A_{33}fe^{-kx})/|D|; \quad (23)$$

$$\overline{\xi} = (A_{14}cos\psi - A_{12}(A_{24}cos\psi + A_{44}sin\psi)/A_{22} + A_{34}fe^{-kx})/|D|. \quad (24)$$

In the works [4-5], considered above in the analysis, data are given on the relationship of the bending parameter $\overline{D}$ with the tension force of the rope in the guides, according to the scheme on Fig. 1, b. Therefore, in the future, the main attention is paid to this scheme.

Table 2 for the load scheme (Fig. 1, b) shows the results of calculations of the relative $\overline{P}1 = P1/P_t$ stretching forces of the ultimate elastic state of the ropes based on formula (16) at the standard bending parameters [2] $b_1(\overline{D}) = 11.2 - 25$ and at $\overline{D} = 120$, as well as the force $\overline{P}3$ corresponding to the scheme of pure stretching [8] (stretching of a straight rope in guides). The characteristics of ropes No. 1–3 are presented in Table 3.

Table 4 shows the relative values $\overline{P}1 = P1/P_t$ for ropes of various designs from Table 3 with bending parameters $\overline{D}$ in the full range $\overline{D} = 10 - 120$. Along with the numerical value of the force $\overline{P}1$, the numbers of the wires that determine the ultimate elastic state of the rope are indicated. For example, in Fig. 2, No. 123 and No. 2 are shaded, and in Fig. 3, No. 2, No. 8, and No. 1 are wires whose deformation (17) determines the ultimate elastic state of the rope at different values of the bending parameter $\overline{D}$. Table 5 refers to the ultimate elastic state of ropes when loaded with a freely suspended load (Fig. 1, a).

Table 2 – Relative stretching forces $\overline{P}1 = P1/P_t$ of ropes in guides (Fig. 1, b) with normative parameters $b_1(\overline{D})$ [2] of bending in winding on a drum

Mechanism class		M1	M2	M3	M4	M5	M6	M7	M8	$\overline{D}$	$\overline{P}3$
№	$b_1(\overline{D})$	11.2	12.5	14	16	18	20	22.5	25	120	
1	Stretching with winding on a drum	0.563	0.585	0.607	0.630	0.648	0.663	0.677	0.690	0.765	0.765
2		0.201	0.257	0.309	0.367	0.412	0.447	0.481	0.512	0.741	0.741
3		0.211	0.265	0.316	0.370	0.412	0.447	0.481	0.510	0.717	0.725

Table 3 – Characteristics of ropes: geometric, strength and rigidity

№	Design designation: 12×(…): 6×(…) – the number of strands; 1/0.6; 6/0.6...//7/1.3...//6/1.5 – the number of wires in the layers of strands and their diameter	d_r, mm	P_t, kN	Stiffness of the section × 10^{-4}			
				G_{11}, N	G_{12}, Nmm	G_{22}, Nmm^2	G_{33}, Nmm^2
1	12×(1/0.6–6/0.6) - 6×(1/0.6+6/0.6)+o.c.	9.3	57.0	515	359.8	620.3	179
2	12×(1/1.4–6/1.4–6(1/0.8+6/0.8+12/0.8)+o.c.	20.5	289.5	2867	45856	19121	4815
3	6×(1/2.4+7/1.8+7/1.8//7/1.3+14/2.2)+ o.c.; GOST 7668–80	39	985.5	8216	48533	292924	5903
4	6×(1/1.2+7/0.9+7/0.9//7/0.65+14/1.1)+ +o.c ;GOST 7668–80	20	246.4	2048	6087	18487	390.4
5	6×(1/1.5+6/1.4+6/1.15//6/1.5)+o.c.; GOST 2688–80	21	267.2	2271	5794	15346	662.7
6	6×(1/2.4+7/1.8+7/1.8//7/1.3+14/2.2) +7×(1/1.7+6/1.6); GOST 7669–80	39	1146	11267	54828	296982	5982
7	6×(1/1.2+6/1.1)+o.c.; GOST 3069–80	10.5	64.25	560	734	1029	70.4
8	6×(1/1.2–6/1.1)+o.c.;GOST 3069–80	10.5	64.25	553	503	523	70.4
9	6×(1/1.2+6/1.1)+ 1× (1/1.2+6/1.1); GOST 3066–80	10	74.95	715	571	602	63.1
10	1/1,15+6/1	3.15	9.2	100	25.8	15.1	25.7
11	1/1.15+6/1+12/1; GOST 3063–80	5.15	24.28	261	123	84.6	22.7
12	1/1.15–6/1+12/1; GOST 3063–80	5.15	24.28	261	72.5	85.8	22.7
13	1+7/2.4+7/1.8//7/1.8+14/2.2; GOST 7668–80	12.7	164	1790	1837	2938	844
14	1/1.9+6/1.8+12/1.8+18/1,8; GOST 3064–80	12.5	151	1472	1618	2563	667
15	1/1.9+6/1.8–12/1.8+18/1.8; GOST 3064–80	12.5	151	1472	630	2503	667
16	1/1.3-6/1.2–(6/1+6/1.3)+18/1.1	7.9	60.63	630	136	443	62.3

Table 4 – Dependence of the tensile strength $\overline{P}1 = P1/P_t$ of the ultimate elastic state of the ropes on the bending parameter $\overline{D}$ when winding on a drum

$\overline{D}$ / № $r.$	10	20	30	40	50	60	70	80	90	100	120	$\overline{P}3$
1	0.537 №123	0.663 №123	0.708 ″	0.723 ″	0.747 ″	0.757 ″	0.765 №2	0.765 ″	0.765 ″	0.765 ″	0.765 ″	0.765 №2
2	0.343 №107	0.547 №107	0.625 ″	0.667 ″	0.692 ″	0.709 №195	0.719 ″	0.726 ″	0.732 №189	0.737 №189	0.741 №116	0.741 116
3	0.149 №139	0.447 ″	0.553 ″	0.607 ″	0.640 ″	0.662 ″	0.678 ″	0.690 ″	0.700 ″	0.707 №139	0.717 №121	0725 №2
4	0.148 №139	0.447 ″	0.554 ″	0.608 ″	0.641 ″	0.663 ″	0.670 ″	0.691 ″	0.701 ″	0.708 №121	0.711 №121	0.728 №2
5	0.141 №51	0.405 №51	0.511 ″	0.567 ″	0.602 ″	0.626 ″	0.643 ″	0.656 ″	0.666 ″	0.674 ″	0.687 ″	0.713 2

$\overline{D}$ / № $r.$	10	20	30	40	50	60	70	80	90	100	120	$\overline{P}3$
6	0.241 №187	0.533 ″	0.636 ″	0.689 ″	0.712 №1	0.712 ″	0.712 ″	0.712 ″	0.712 ″	0.712 ″	0.712 ″	0.712 ″
7	0.405 № 27	0.580 ″	0.642 ″	0.673 ″	0.693 ″	0.706 ″	0.716 ″	0.723 ″	0.728 ″	0.733 ″	0.738 №2	0.738 №2
8	0.408 №3	0.584 ″	0.647 ″	0.679 ″	0.698 ″	0.711″	0.721 ″	0.728 ″	0.730 №2	0.730 ″	0.730 ″	0.730 ″
9	0.483 №2	0.575 ″	0.608 ″	0.625 ″	0.635 ″	0.642 ″	0.647 ″	0.651 ″	0.653 ″	0.656 ″	0.659 ″	0.677 №1
10	0 №2	0.161 №2	0.391 ″	0.509 ″	0.581 ″	0.630 ″	0.666 ″	0.691 ″	0.711 ″	0.728 ″	0.753 ″	0.788 №1
11	0.041 №2	0.432 №2	0.572 ″	0.644 ″	0.687 №8	0.715 ″	0.736 ″	0.751 ″	0.763 ″	0.772 ″	0.778 № 1	0.778 №1
12	0.041 №2	0.432 №2	0.572 ″	0.644 ″	0.687 №8	0.715 ″	0.736 ″	0.751 ″	0.763 ″	0.772 ″	0.778 №1	0.778 №1
13	0 № 23	0.266 №23	0.465 ″	0.567 ″	0.626 ″	0.670 ″	0.700 ″	0.722 №9	0.734 ″	0.743 ″	0.758 ″	0.789 №1
14	0 №8	0.227 №8	0.400 ″	0.490 ″	0.544 ″	0.580 ″	0.607 ″	0.626 ″	0.642 ″	0.654 ″	0.673 ″	0.711 №1
15	0 №8	0.227 №8	0.400 ″	0.490 ″	0.544 ″	0.580 ″	0.607 ″	0.626 ″	0.642 ″	0.654 ″	0.673 ″	0.711 №1
16	0.694 №1	….	….	….	….	….	….	….	….	….	….	0.694 №1

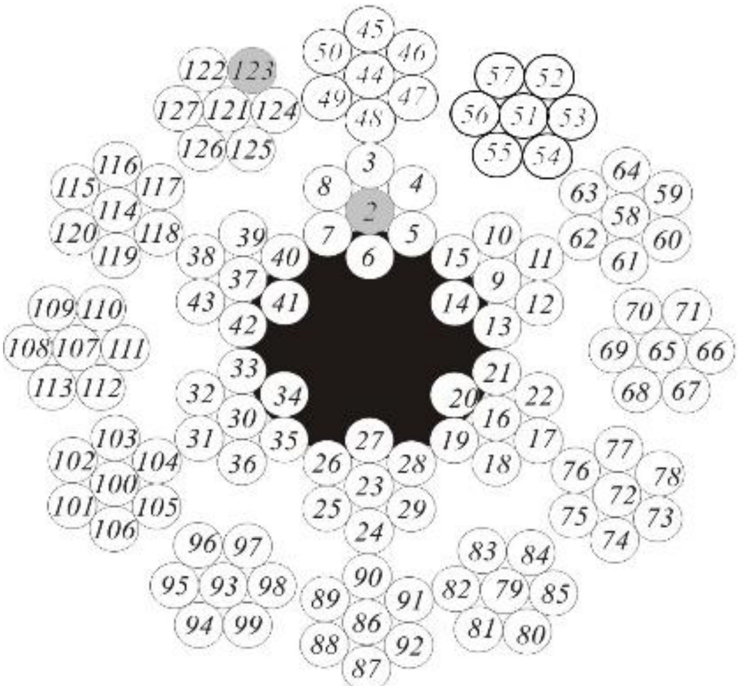

Fig. 2. Rope No. 1: 12(1–6) – 6(1+6) +o.c

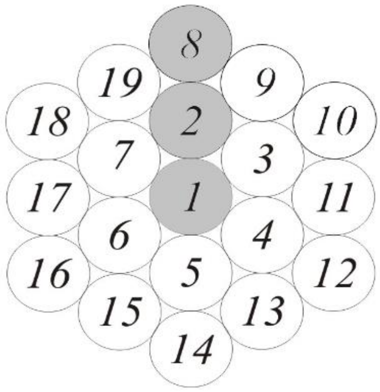

Fig. 3. Rope No. 11: 1+6+12

Table 5 – Dependence of tensile forces $\overline{P}2 = P2/P_t$ of the ultimate elastic state of the ropes on the parameter $\overline{D}$ when the load is freely suspended

$\overline{D}$ / № $r.$	10	20	30	40	50	60	70	80	90	100	120	$\overline{P}4$
11	0.02 №2	0.243 №1	″	″	″	″	″	″	″	″	0.243 ″	0.243 №1
12	0.03 №2	0.263 №2	0.347 ″	0.389 ″	0.416 ″	0.433	0.446 ″	0.455 ″	0.463 ″	0.469 ″	0.471 ″	0.523 №2
16	0.04 №2	0.593 №8	″	″	″	″	0.592	″	″	0.592	″	0.592

Let us consider the factors that affect the deformation components (17). The first component $\overline{\varepsilon}K_{e\varepsilon_i}$ is the stretching deformation of the wires, which is related to the stretching deformation ε of the rope. The second component $\pm\overline{\theta}K_{e\theta_i}$ is related to the deformation θ of the twisting of the rope ("$+$" sign for layers of wires, the direction of twisting of which coincides with the direction of twisting θ and the opposite sign is "-"). When the rope is stretched in the guides, regardless of the construction of the rope, the torsional deformation $\theta = 0$ component $\overline{\theta}K_{e\theta_i} = 0$. When the load is freely suspended (Fig. 1, a) the component $\overline{\theta}K_{e\theta_i}$ depends on the construction of the rope.

In the case of free stretching of a straight rope, the constructive sign of the absence of torsional deformation θ is the zero value of impact stiffness, $G_{12} = 0$. The degree of torsion of the rope is determined by the parameter [8]: $\psi = G_{12}/maxG_{12} \leq 1$, where $maxG_{12}$ – is the maximum possible value for this type of rope construction. So, for the rope No. 11 of construction 1+6+12 $\psi = 1$ and for the rope No. 12 of construction 1– 6+12 $\psi = G_{12}/maxG_{12} = 725/1230 = 0.589$.

Tables 4 and 5 and Fig. 6 show a significant influence of the rope design (partly parameter) on the force $\overline{P}2$ of the limit elastic state with a freely suspended load. For the same ropes, the data in Table 4 and Figs. 4 and 5 show the uniformity of the force $\overline{P}1$ of the ultimate elastic state with the load in the guides.

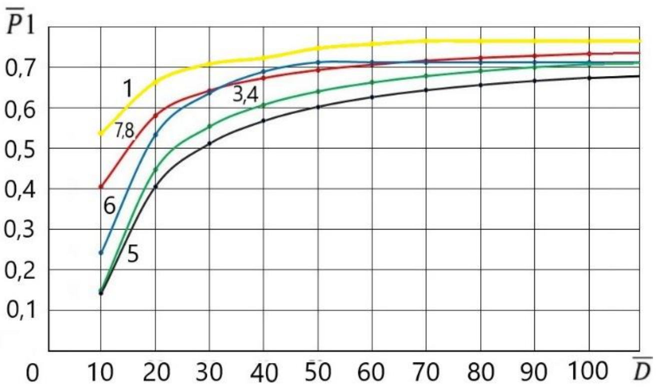

Fig. 4. Dependence $\overline{P}1(\overline{D})$ of double twist ropes

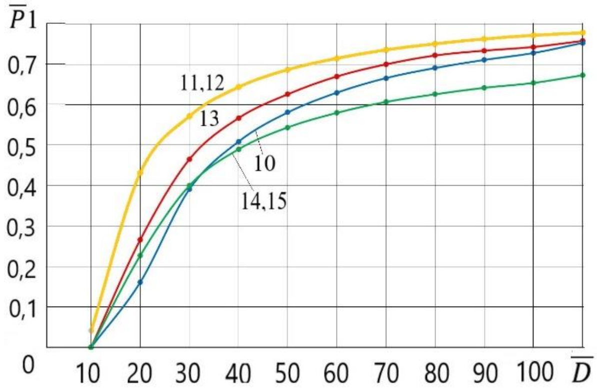

Fig. 5. Dependence $\overline{P}1(\overline{D})$ of single twist ropes

The component $\overline{\chi}K_{e\chi_i}$ in the specific strain (17) represents the stretching from the bending strain $\overline{\chi}$ of the rope. This component is given by the parameter $\overline{D}$ and is related to interelement friction. The mechanics of the formation of this deformation is described in [6]:

$$K_{e\chi} = \nu cos^2\alpha \, cos\varphi, \quad (25)$$

where $\nu(\overline{D})$ – is the coefficient due to interelement friction.

We accept on the basis of [9]: $\nu = 0.2 - 0.1$ respectively for the wires of the inner and outer layers and $\nu = 0$ for the central wires.

Somewhat anomalous values of forces $\overline{P}1$ and $\overline{P}2$ are associated with this feature ($\nu = 0$) their constant values, independent of the bending parameter $\overline{D}$. Table 4 shows several calculation results of this feature. Thus, for the rope No. 1 at $\overline{D} = 70 - 120$ the independence of the force from the bending parameter $\overline{P}1 = 0.765$ due to the fact that the limit state is determined by the central wire No. 2, Fig. 3, for which $\nu = 0$. A similar situation for the rope No. 8: in the interval $\overline{D} = 90 - 120$ the force of the ultimate elastic state under load in the guides is unchanged ($\overline{P}1 = 0.730$).

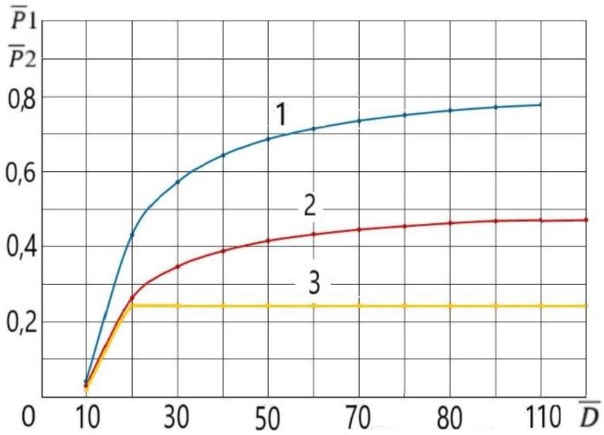

Fig. 6. Dependence $\overline{P}1(\overline{\mathrm{D}})$ and $\overline{P}1(\overline{\mathrm{D}})$ of ropes 1+6+12 and 1– 6+12:
1 – ropes №11, 12 force $\overline{P}1$; 2 – rope №12, force $\overline{P}2$; 3 – rope №11, force $\overline{P}2$

The same peculiarity is manifested with a freely suspended load. As an example, this is shown in Table 5 for the rope No. 11 at $\overline{D} = 20 - 120$ ($\overline{P}2 = 0.243$), due to the fact that the limit state is determined by the central wire No. 1, which does not receive stretching from the bending of the rope, $\nu = 0$.

The component $\overline{\xi} K_{e\xi_i}$ in the specific deformation (17) should take into account the stretching of the wires from the bending deformation $\overline{\xi}$ of the rope in the plane orthogonal to the winding on the drum. Such a deformation occurs with asymmetry in the cross-section of the rope, when its impact stiffnesses G_{14}, G_{24}, G_{34} are not zero. This happens if the wires are broken. This work does not consider such a state.

Fig. 4 shows a graphical representation of the dependence $\overline{P}1(\overline{\mathrm{D}})$ for the double-twist ropes specified in Table 3 (numbers: 1; (7. 8); 6; (3, 4), 5). Fig. 5 shows a similar representation for single winding ropes (numbers: (11, 12); 13; 10; (14, 15)).

Fig. 6 shows the dependences $\overline{P}1(\overline{\mathrm{D}})$ and $\overline{P}2(\overline{\mathrm{D}})$ of the limit elastic state for load schemes in guides (Fig. 1, b) and free suspension of the load (Fig. 1, a). The graphs are constructed for ropes of construction 1+6+12 and 1–6+12 (No. 11 and No. 12 according to Table 3).

Conclusions. Analysis of Table data 2 and 4 graphs of Fig. 4 – 6 leads to the following conclusions.

For ropes of the same design, the relative tensile forces under loading in the guides $\overline{P}1$ of the limit elastic state are practically the same.

Efforts $\overline{P}1$ differ significantly for different designs. For double-twisted ropes, at $\overline{D} = 10$ the greatest force value $\overline{P}1$ has construction 12(1–6)±6(1+6)+o.c., GOST 7681–80 (No. 1 according to Table 3). The effort $\overline{P}1$ of this design is 3.8 times greater compared to designs No. 3, 4, 5 (GOST 7668–80 and GOST 2688–80). For other structures (No. 6 according to Table 3, GOST 7668–80 and No. (7, 8) GOST 3069–80), the forces differ by 2.2 and 1.3 times, respectively. For other structures (No. 6 according to Table 3, GOST 7668–80 and No. (7, 8) GOST 3069–80), the forces differ by 2.2 and 1.3 times, respectively. As the bending parameter $\overline{D}$ increases, the indicated ratios decrease. So, at $\overline{D} = 20 - 120$ the effort ratio $\overline{P}1$ of the specified structures being only 1.48-1.07.

With the bending parameter $\overline{D} \geq 120$ the limit elastic force $\overline{P}1$ (according to the load scheme in the guides (Fig. 1, b)) is practically equal to the force $\overline{P}3$ of the loaded straight rope in the guides (pure stretching).

We believe that this theoretical study reveals the practical validity of the recommendations in Germany [5] regarding the assignment of the parameter $\overline{D} \geq 100 - 120$ for mine installations in order to increase the service life of ropes.

Among single twist ropes, the design 1+6+12, GOST 3063–80 (No. 11 and No. 12 according to Table 3) shows the greatest effort $\overline{P}1$. In comparison with the design 1/1.9+6/1.8+12/1.8+18/1.8; GOST 3064–80 and construction 1+7/2.4+7/1.8//7/1.8+14/2.2 (No. 14 and No. 13 according to Table 3) the ratio of efforts $\overline{P}1$ at $\overline{D} = 20$ is 1.9-1.6. As the bending parameter $\overline{D}$ increases, these ratios decrease. So, when $\overline{D} = 120$ the effort ratio $\overline{P}1$ is 1.15-1.03.

The smallest possible (suitable) values of the bending parameter for double-twisted ropes $\overline{D} = 10$ and for single-twisted ropes $\overline{D} = 15$.

When stretching with bending in the guides, the forces $\overline{P}1$ as well as during pure stretching $\overline{P}3$ are the same for single-sided and cross-winding ropes (No. 11 and No. 12 according to Table 3).

The tensile strength of the ultimate elastic state when the load is freely suspended is much smaller compared to the load scheme in the guides, especially for torsion ropes, which makes such structures unsuitable for such a load scheme.

Ropes with a metal core have a stress of ultimate elastic state $\overline{P}1$ 15-17% lower compared to similar designs of ropes with organic cores.

Twisted ropes show the forces of the elastic limit state $\overline{P}2$ when the load is freely suspended by 1.7–2.9 times less compared to the force $\overline{P}1$ when loaded in the guides. This indicator is extremely important for the practice of using ropes.

We believe that the research data are appropriate in solving the issue of building a methodology for calculating the static strength of lifting ropes based on the characteristics of their ultimate elastic state, which will ensure stable optimality of the use of ropes.

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ЗАЛЕЖНІСТЬ ЗУСИЛЛЯ РОЗТЯГАННЯ ГРАНИЧНОГО ПРУЖНОГО СТАНУ КАНАТІВ ВІД ПАРАМЕТРА ЗГИНАННЯ НА БАРАБАНІ

[1]**Вовк П.Є.,** аспірат,
vovk.pavel.1995@gmail.com, ORCID: 0000-0001-6156-1686
[1]**Чаюн І.М.**, д.т.н., професор,
i.m.c@ukr.net, ORCID: 0000-0003-0867-8791
[1]*Національний університет «Одеська політехніка»*
пр. Шевченко 1, Одеса, 65044, Україна

Анотація. На основі раніше розробленого методу аналітичного визначення граничного пружного стану канатів досліджена залежність $P(\overline{D})$ зусилля розтягання з навиванням каната на барабан від параметра $\overline{D} = D_d/d_r$ (відношення діаметрів барабана і каната) в інтервалі $\overline{D} = 10 - 120$. Залежіть $P(\overline{D})$ визначалась для двох схем навантаження канату: розтягання канату з навиванням на барабан вільно підвішеним вантажем та розтягання канату з навиванням на барабан вантажем в напрямних. На основі розробленого методу представлено залежність $P(\overline{D})$ для 16-ти кантаів різних конструкцій. Зусилля розтягання представлено в відносній формі $\overline{P}(\overline{D}) = P(\overline{D})/P_t$ (P_t – сумарне розривне зусилля дротів канату). Зусилля $\overline{P}(\overline{D})$ суттєво залежить від конструкції канату і параметра $\overline{D}$ згинання. На ділянці $40 \leq \overline{D} \leq 120$ зусилля$\overline{P}(\overline{D})$ монотонно практично лінійно зростає доходячи до значення відповідного розрахунковій схемі розтягання прямого канату. Для різних конструкцій при параметрі $\overline{D} = 40$ зміна зусилля $\Delta\overline{P} = 0{,}723 - 0{,}578 = 0{,}145$ при $\overline{D} = 120$ зусилля $\Delta\overline{P} = 0{,}765 - 0{,}677 = 0{,}08$. На ділянці $40 \geq \overline{D} \geq 20$ залежність $\overline{P}(\overline{D})$ не лінійна, при $\overline{D} = 40$ зусилля $\Delta\overline{P} = 0{,}663 - 0{,}418 = 0{,}245$. Ділянка $20 \geq \overline{D} \geq 10$ характерна різкою зміною зусилля $\overline{P}(\overline{D})$, при параметрі згинання $\overline{D} = 10$ зусилля $\Delta\overline{P} = 0{,}416$ Для більшості конструкцій канатів при $\overline{D} < 10$ зусилля $\overline{P}(\overline{D})$ близькі до нульових значень. При розтяганні вільно підвішеним вантажем мало крутних канатів зусилля $\overline{P}(\overline{D})$ в 1,6 –1,7 разів менші в порівнянні з розтяганням в напрямних. Для канатів крутних (одностороннього звивання) співвідношення складає 2,5–3,4 рази. В нормативних методиках розрахунків піднімальних канатів використовуються характеристики P_t або $P_a = 0{,}83P_t$, які не враховують особливості деформування і конструкції канатів. Вважаємо, що наведена інформація є доцільною в вирішенні питання побудови методики розрахунку піднімальних канатів на статичну міцність на основі характеристик їх граничного пружного стану, що забезпечить стабільну оптимальність використання канатів, дозволить раціонально обирати вид конструкції канату та його розміри.

Ключові слова: канат, міцнісні характеристики, розрахункова схема, параметр згинання при навиванні на барабан, методика розрахунку на статичну міцність.

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