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ANALYSIS OF DYNAMICS AND OPTIMISATION OF TRANSIENT MOTION MODES OF HOISTING MECHANISMS OF OVERHEAD URBAN CRANES

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Abstract. The research of operating modes of hoisting mechanisms and machines, in particular, overhead and city-building cranes is carried out. Based on dynamic analysis of operating modes of mechanisms of the mentioned machines the character of load occurrence in their elastic elements (ropes) is established. The synthesis of optimum transient modes of operation (start/braking) of city-building and overhead cranes is carried out. The models of L.D. Landau and N.A. Lobov for physical and mechanical analysis of motion of the specified machines are used, on the basis of which with attraction of methods of classical calculus of variations the laws of motion of a cargo and a cargo carriage of similar cranes which optimize (minimize) inevitably arising pendulum oscillations of a cargo on a rope (as in the period of start-up of the crane, and at its braking, up to full stop) are received analytically.

The evaluation of motion modes in the work is carried out by using a criterion with sub-integral functions in the form of "energy" of accelerations of higher orders (third, fourth, etc.). Exactly such criteria are used in the selection of modes of motion of lifting mechanisms of cranes of bridge/bridge-building type, which have in their composition elastic elements (ropes). The above approach makes it possible to achieve continuity of functions of velocities and accelerations of various links of the system, which leads to a significant reduction in their oscillations. The analysis of modes of motion of the system, which is described by the main coordinate of motion and elastic coordinates, allowed one to establish the regularity of using the proposed criterion to minimize the elastic vibrations of individual links (ropes) of the crane hoisting mechanism.

The results of this study can be further used to clarify and improve existing engineering methods for calculating loads in flexible/elastic elements (ropes) of bridge and city-building cranes both in the design of hoisting mechanisms and in the modes of their real operation when mechatronic control systems are used in controlling the movement of the load carriage.

Key words: analysis, dynamics, optimization, transient motion modes, starting, braking, hoisting mechanisms, overhead and urban cranes.

Introduction. The nature of movement of the main links of lifting and transporting mechanisms and machines significantly affects the quality of the performance of technological processes by such machines and mechanisms, their productivity, reliability, durability, etc. Thus, the law of change determines the mode of movement of this or that link of the mechanism or machine in time or position of kinematic characteristics (displacements, speeds, etc.) and time of movement. For a certain design of a hoisting crane mechanism, the law of motion of an individual link (for example, the drive of a city building or bridge crane) determines the law of motion of other links. The complete time of motion of a hoisting mechanism is the time interval from the beginning of motion to its end. It is the complete cycle of motion of a hoisting crane, conveying machine or mechanism can be divided into three main stages: 1) stage of start (run-up, acceleration); 2) stage of steady motion (with constant speed; 3) stage of stop (run-out). Each of the above stages is characterized by the duration in time and the laws of motion of the links. In this study, we will assume that the durations of the stages considered are set, and it is necessary to determine on them such modes of motion that minimize certain criteria of quality of motion or the hoisting/transporting mechanism or machine itself.

In particular, the start-up stage of a hoisting mechanism, transporting machine is characterized by the growth of the speed of a link, for example, a drive, from zero value to a certain value. In this case, various modes of motion minimizing inertial loads, the power of the drive itself, loads of oscillatory nature (on elastic elements - ropes of hoisting mechanisms of cranes, for example, of the city-building or bridge types) or a complex of such characteristics can be used. In addition, at the stage of start-up can be found modes of motion, which at given restrictions on the above characteristics provide, in particular, and the minimum duration of start-up.

This study is devoted to the solution of the problems outlined above.

Analysis of publications on the topic of research. The authors [1-21] have investigated in sufficient detail the main features of the dynamics of the functioning of overhead/bridge cranes and have given scientifically justified conditions and laws of motion of the hoisting carriage and the load on the crane rope, under which the arising pendulum vibrations of the load can be minimized, however, the results obtained in the cited works do not always have a justified physical meaning and can be implemented in the real operation of hoisting mechanisms, since they are based on incorrect initial and final (terminal) conditions. In particular, usually the works cited above consider only kinematic conditions of motion, both at the beginning and at the end of the transient period (say, the start-up mode). There is a problem with this approach, since the terminal conditions (at the beginning of the motion) lack the cause that led to the motion - the force factor. Also, after the end of the start-up period, when the crane system enters the steady state (and begins to move at a constant speed), based on physical considerations, Newton's first law must be fulfilled, and the force factor is absent, or the equidistance of all forces is zero. Among the published and cited above works, the authors of this study have nowhere (not once!) met the confirmation of these relevant reasoning. It should also be noted that when considering the forces of resistance to movement, which are reduced in the start-up period of the crane system, as a rule. to the forces of dry friction, the horizontal component of these forces is considered only in one phase, when only the movement from the place begins (ie, the crane trolley ahead of the movement of the load on the rope), but the start-up period can last for a period of time when the sign of this force is reversed (the load on the rope pulls the trolley). Therefore, the generalized dry friction force can lead to chaotic oscillations of the system (dynamic, deterministic chaos). This is confirmed by numerical analysis of the problem on a PC computer. Consequently, when correctly formulating the problem, it is necessary to clearly limit exactly what time interval is described by the proposed model equations of motion. It is these above-mentioned circumstances that force the authors of this study to correct the results previously obtained in the above-quoted works, to adjust them to satisfy the generally recognized laws of classical mechanics.

Goal and task lie in the substantiation of the physical and mechanical model and laws of motion of the cargo trolley and the cargo on the rope of overhead and city-building cranes, at which the pendulum oscillations of the cargo in the transient modes of operation of these cranes can be minimized. At the same time, two basic models of motion of mechanical system (model of Lobov N.A. [3] and model of Landau L.D. [1]) are used to achieve the purpose of the research. The author's model [3] is linear, and the model of work [2] is linearized in this study.

Research data and methodology. The methods of mathematical modelling, classical calculus of variations and mathematical physics are used in this work. In addition, the results of works of domestic and foreign scientists on the methodology of research, analysis and synthesis of optimal transient motion modes (starting, braking, reversing) of hoisting mechanisms of urban/bridge cranes have been used.

Research outcomes.

1. Calculation of optimal modes of motion of the hoisting mechanism of the bridge/bridge crane in the framework of the model of Lobov N.A. [3].

1.1. *The starting mode of the hoisting mechanism in which pendulum oscillations are minimised.* As an analogue scheme of pendulum oscillations of the load, we use the dynamic system of work [3], shown in Fig. 1. The designations of the above model are retained in this study.

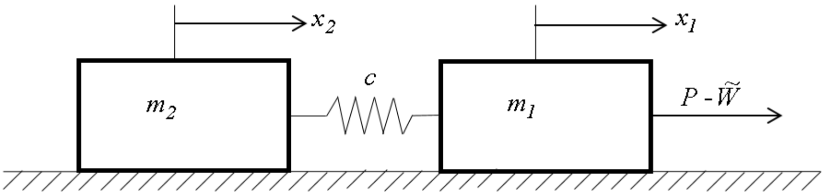

Fig. 1. Analogue circuit of pendulum oscillations of the crane load [2, 7]

In Fig. 1, the following notations are introduced: m_1 – the mass of the crane or load trolley given to the translational motion of the crane or trolley; m_2– mass of the load; $c = m_2 \cdot g$ – weight of the load; g – free fall acceleration $\left(g = 9{,}81\, м/c^2\right)$; P – total traction or braking force of the crane or trolley drive wheels; $\tilde{W}$ – the force of resistance to the movement of the crane or trolley (non-linear) and is described by the Coulomb-Amontons law (dry friction), $\tilde{W} = W \cdot sign(\dot{x}_1)$, W – amplitude of dry sliding friction force; x_1 and x_2 – horizontal displacement of masses m_1 and m_2; S – total cable tension; φ – angle of deflection of the ropes from the vertical; T – horizontal component of forces in ropes; H – the length of the cables' tangs; c – cable rigidity (horizontal) $\left(c = m_2 g / H\right)$.

Since the maximum deviations of the cables from the vertical do not exceed $(10\ldots12)^o$, let us assume that $sin\varphi \approx \varphi,\ cos\varphi \approx 1{,}0$. Having made that assumption [2]:

$x_2 = x_1 + H \cdot \varphi,\ s = G = m_2 g$, and the horizontal component of rope tension has the form of:

$$\mathrm{T} = \mathrm{s} \cdot \varphi = \mathrm{m}_2 g \cdot (x_2 - x_1)/H . \quad (1)$$

Equation of motion of a crane or trolley(mass m_1) assumes the following form:

$$m_1 \cdot \ddot{x}_1 + \frac{m_2 g}{H} \cdot (x_1 - x_2) = P - \tilde{W} = P - W \cdot sign(\dot{x}_1), \quad (2)$$

a equation of motion of the load in the horizontal direction (for mass m_2):

$$m_2 \cdot \ddot{x}_2 + \frac{m_2 g}{H} \cdot (x_2 - x_1) = 0 . \quad (3)$$

In the scheme shown in Fig. 1, within the framework of the given dynamic model of the system, the mass m_2 slides along the supporting surface without friction [2, 7]. The movement of such a mechanical system is described by the following equations:

$$\begin{cases} m_1 \cdot \ddot{x}_1 + c \cdot (x_1 - x_2) = P - W \cdot sign(\dot{x}_1); \\ m_2 \cdot \ddot{x}_2 + c \cdot (x_2 - x_1) = 0. \end{cases} \quad (4)$$

Comparing (4) with (2) and (3), we see that both systems of equations are identical if we assume $c = m_2 \cdot g / H$. It follows that the dynamic effect of an oscillating load on a crane (or trolley) is similar to that of a load attached by springs with stiffness of (c), that equals G / H [2, 7]. It is this analogy that allows us to visualize the effect of a swinging load on the motion of the crane. When $x_1 > x_2$, the deflected load increases the resistance forces to the crane's movement.

Coefficient $c = G/H$ can be called an analogue of the transverse rigidity coefficient of cables. Indeed, when $x_1 > x_2$:

$$m_1 \cdot \ddot{x}_1 = P - W \cdot sign(\dot{x}_1) - c \cdot (x_1 - x_2) = P - \{W \cdot sign(\dot{x}_1) + c \cdot (x_1 - x_2)\}, \quad (5)$$

and the total drag force (friction + the effect of the load on the rope) will be:

$$F_{onopy} = W \cdot sign(\dot{x}_1) + c \cdot (x_1 - x_2), \quad (6)$$

with $\dot{x}_1 > 0$ from (6) it is obvious that in the case of $x_1 > x_2$ drag force increases.

When $x_2 > x_1$ in (6), as long as $\dot{x}_1 > 0$ it is obvious, that the total drag force decreases.

During the period of crane acceleration we consider that $\dot{x}_1 > 0$, then instead of system (4) we have:

$$\begin{cases} m_1 \cdot \ddot{x}_1 + c \cdot (x_1 - x_2) = P - W; \\ m_2 \cdot \ddot{x}_2 + c \cdot (x_2 - x_1) = 0. \end{cases} \quad (7)$$

By simple transformations and by introducing substitution of variables on $\xi = (x_1 - x_2)$ the system (7) can be reduced to:

$$\ddot{\xi} + \left(1 + \frac{m_2}{m_1}\right) \cdot \frac{g}{H} \cdot \xi = \frac{P - W}{m_1}, \quad (8)$$

else:

$$\ddot{\xi} + \Omega^2 \cdot \xi = \frac{P - W}{m_1}, \quad \Omega^2 = \left(1 + \frac{m_2}{m_1}\right) \cdot \frac{g}{H}. \quad (9)$$

As initial conditions for equation (9) we choose the following:

$$x_1\big|_{t=0} = x_2\big|_{t=0} = 0 \Leftrightarrow \xi\big|_{t=0} = 0; \quad \dot{x}_1\big|_{t=0} = \dot{x}_2\big|_{t=0} = 0 \Leftrightarrow \dot{\xi}\big|_{t=0} = 0. \quad (10)$$

Then the solution of equation (9) under initial conditions (10) has the form:

$$\xi(t) = \left(\frac{P - W}{m_1}\right) \cdot \frac{1}{\Omega^2} \cdot (1 - cos\Omega t) = \left(\frac{P - W}{m_1}\right) \cdot \frac{2}{\Omega^2} \cdot sin^2\left(\frac{\Omega t}{2}\right). \quad (11)$$

The modulus of the tension force of the rope with the load (in the horizontal direction) acquires a value:

$$|T| = c \cdot |x_1 - x_2| = c \cdot \left(\frac{P - W}{m_1}\right) \cdot \frac{2}{\Omega^2} \cdot sin^2\left(\frac{\Omega t}{2}\right) = \frac{m_2 \cdot g}{H} \cdot \left(\frac{P - W}{m_1}\right) \cdot \frac{2}{\left(1 + \frac{m_2}{m_1}\right) \cdot \frac{g}{H}} \cdot sin^2\left(\frac{\Omega t}{2}\right) =$$

$$= 2 \cdot \frac{m_2}{m_1} \cdot (P - W) \cdot \frac{1}{\left(1 + \frac{m_2}{m_1}\right)} \cdot sin^2\left(\frac{\Omega t}{2}\right) = 2 \cdot \frac{m_2}{(m_1 + m_2)} \cdot (P - W) \cdot sin^2\left(\frac{\Omega t}{2}\right). \quad (12)$$

At the time t_n^*, the following can be deduced:

$$\frac{\Omega \cdot t_n^*}{2} = \frac{\pi}{2} \cdot (2n - 1), \quad n \in N, \quad (13)$$

significant overloading (force, dynamic) of the crane rope system occurs because:

$$|T|\big|_{t = t_n^*} = 2 \cdot \frac{m_2}{(m_1 + m_2)} \cdot (P - W). \quad (14)$$

Such situations in the operation of hoisting mechanisms of overhead travelling cranes and urban construction cranes are undesirable, as such overloads of ropes may lead to rope breaks and consequently to accidents. Therefore, it is desirable to avoid such states of the hoisting system/mechanisms of the above cranes.

Let's determine the mode of motion of the crane hoisting mechanism (trolley and load), at which the force and dynamic loads on the rope will be minimal during the start-up period of the crane system. Duration of the start-up period of the hoisting mechanism is denoted by τ_s. The criterion of the quality of movement, at which the start of the crane load lifting mechanism creates minimum loads on its (crane) rope system, takes the following form:

$$I = \left\{ \frac{1}{\tau_s} \cdot \int_0^{\tau_s} (c \cdot \xi)^2 \, \mathrm{dt} \right\}^{\frac{1}{2}} \Rightarrow \min . \tag{15}$$

Using equation (9), the criterion (15) can be represented in the following form:

$$I = \left\{ \frac{1}{\tau_s} \cdot \int_0^{\tau_s} \left[c \cdot \left(\frac{P - W}{m_1} - \ddot{\xi} \right) \cdot \frac{1}{\Omega^2} \right]^2 \mathrm{dt} \right\}^{\frac{1}{2}} \Rightarrow \min . \tag{16}$$

The necessary condition for realization/achievement of criteria (15), (16) is the Euler-Poisson equation [9, 12, 13], which in this case (for constant, independent of time t, values P and W assumes the form:

$$\xi^{(IV)} = 0 . \tag{17}$$

We will look for the solution of equation (17) in the form of t a third-order spline:

$$\xi(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3 , \tag{18}$$

where constants a_0, a_1, a_2, a_3 are found from the following terminal conditions:

$$\xi\big|_{t=0} = 0 ; \quad \dot{\xi}\big|_{t=0} = 0 ; \quad \ddot{\xi}\big|_{t=0} = \frac{P - W}{m_1} ; \quad \dot{\xi}\big|_{t=\tau_s} = 0 . \tag{19}$$

Applying the coefficients a_0, a_1, a_2, a_3 obtaining:

$$a_0 = 0 ; \quad a_1 = 0 ; \quad a_2 = \frac{(P - W)}{2 \cdot m_1} ; \quad a_3 = -\frac{(P - W)}{3 \cdot \tau_s \cdot m_1} . \tag{20}$$

Thus for $\xi(t)$, that satisfies motion quality criteria (15), (16), one would obtain:

$$\xi(\mathrm{t}) = \frac{(P - W)}{2 \cdot m_1} \cdot t^2 - \frac{(P - W)}{3 \cdot \tau_s \cdot m_1} \cdot t^3 = \frac{(P - W)}{m_1} \cdot \left\{ \frac{\mathrm{t}^2}{2} - \frac{t^3}{3 \cdot \tau_s} \right\}, \quad 0 \le t \le \tau_s . \tag{21}$$

With $\xi(t)$, (21) for $|T|$ one has:

$$|T| = c \cdot |\xi| = c \cdot \frac{(P - W)}{m_1} \cdot \left\{ \frac{\mathrm{t}^2}{2} - \frac{t^3}{3 \cdot \tau_s} \right\}, \quad 0 \le t \le \tau_s . \tag{22}$$

The minimal value $|T|$ reaches at the time $t = 0$ (at the start), and it reaches its maximal value $t = \tau_s$:

$$|T|_{\max} = |T|\big|_{t=\tau_s} = c \cdot \frac{(P - W)}{m_1} \cdot \left\{ \frac{\tau_s^2}{2} - \frac{\tau_s^3}{3 \cdot \tau_s} \right\} = c \cdot \frac{(P - W)}{m_1 \cdot 6} \cdot \tau_s^2 . \tag{23}$$

Then,

$$|T|_{\max} = |T|\big|_{t=\tau_s} = \frac{\mathrm{m}_2 g}{H} \cdot \frac{(P - W)}{6 \cdot m_1} \cdot \tau_s^2 . \tag{24}$$

Graph, that reflects the relationship between $|T|$ and t shown on Fig. 2, has smooth (rather then sharp) character of change with time.

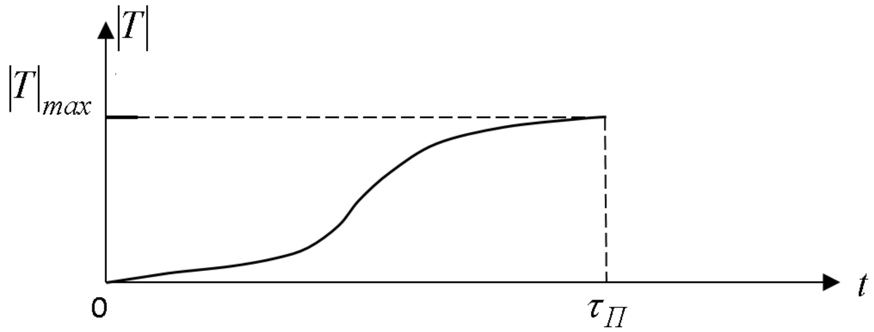

Fig. 2. Dependence $|T|$ on t the optimal starting mode of the crane system

Let's find out what the laws of motion should be with $x_1(t)$ and $x_2(t)$, so that the motion quality criteria (15), (16) under initial conditions are satisfied:

$$x_1\big|_{t=0} = x_2\big|_{t=0} = 0; \quad \dot{x}_1\big|_{t=0} = \dot{x}_2\big|_{t=0} = 0. \tag{25}$$

In order to find $x_2(t)$ let's apply the condition (25) and the second equation from the system (7), obtaining:

$$\ddot{x}_2 = \frac{c}{m_2} \cdot \xi(t). \tag{26}$$

Integrating twice (26) over t (accounting for (25)), one gets:

$$x_2(t) = \frac{c}{m_1 \cdot m_2} \cdot (P - W) \cdot \left\{ \frac{t^4}{24} - \frac{t^5}{60 \cdot \tau_s} \right\}. \tag{27}$$

Since $\xi(t) = x_1 - x_2$, one has:

$$x_1(t) = x_2(t) + \xi(t), \tag{28}$$

$$x_1(t) = \frac{c}{m_1 \cdot m_2} \cdot (P - W) \cdot \left\{ \frac{t^4}{24} - \frac{t^5}{60 \cdot \tau_s} \right\} + \frac{(P - W)}{m_1} \cdot \left\{ \frac{t^2}{2} - \frac{t^3}{3 \cdot \tau_s} \right\}. \tag{29}$$

therefore it is evident that $x_1(t)$ (29) and $x_2(t)$ (27) apply only to the time period $t \in [0, \tau_s]$.

Using mechatronic control system of the crane trolley drive frequency (m_1), it is possible to establish exactly during the start-up of the crane system on the time interval $t \in [0, \tau_s]$ the laws of motion of the load (27) and the load trolley (29) [7].

Noted, the equation $|T|_{max}$ (24) with $|T|_{max}$ (14) demonstrates:

$$\tau_s < \sqrt{\frac{12H}{g \cdot \left(1 + \frac{m_2}{m_1}\right)}}, \tag{30}$$

in the mode of motion according to laws (27), (29) smaller values of dynamic loads are achieved with the cable system of the crane. For $\mathrm{H} = 10\mathrm{m}$, $\frac{m_2}{m_1} \approx 0{,}5$ the value τ_s for (30) gives $\tau_s < 3s$.

1.2 The mode of coasting (braking) of hoisting mechanism at which pendulum oscillations are minimized. By analogy with paragraph 1.1. analyze and optimize the mode of motion of the hoisting mechanism of the crane, at which minimize the pendulum oscillations of the load on the rope, which corresponds to the braking of the crane to a complete stop (run-out mode of the speed of rotation of the drive of the load carriage).

Instead of P one enters (8) $\mathrm{P_B}$ – braking force (which is a constant value independent of time t. Let us denote the duration of the braking process as τ_B.

To fulfill the traffic quality criterion:

$$I^{*}=\left\{\frac{1}{\tau_B}\cdot\int_0^{\tau_B}\left[c\cdot\left(\frac{P_B-W}{m_1}-\ddot{\xi}\right)\cdot\frac{1}{\Omega^2}\right]^2\mathrm{dt}\right\}^{\frac{1}{2}}\Rightarrow\min, \tag{31}$$

a necessary condition for the realization of which is the Euler-Poisson equation (17), we search for the solution of (last) in the form of a spline t of the third order:

$$\xi(t)=\tilde{a}_0+\tilde{a}_1\cdot t+\tilde{a}_2\cdot t^2+\tilde{a}_3\cdot t^3, \tag{32}$$

where constants $(\tilde{a}_0,\tilde{a}_1,\tilde{a}_{2,}\tilde{a}_3)$ are found from the following terminal conditions:

$$\xi\big|_{t=0}=0;\quad \dot{\xi}\big|_{t=0}=0;\quad \ddot{\xi}\big|_{t=0}=\frac{P_B-W}{m_1};\quad \dot{\xi}\big|_{t=\tau_B}=0. \tag{33}$$

Applying (32) for coefficients $\tilde{a}_0,\tilde{a}_1,\tilde{a}_{2,}\tilde{a}_3$ one has:

$$\tilde{a}_0=0;\quad \tilde{a}_1=0;\quad \tilde{a}_2=\frac{(P_B-W)}{2\cdot m_1};\quad \tilde{a}_3=-\frac{(P_B-W)}{3\cdot\tau_\Gamma\cdot m_1}. \tag{34}$$

Therefore, with $\xi(t)$ (32) one gets:

$$\xi(\mathrm{t})=\frac{(P_B-W)}{m_1}\cdot\left\{\frac{\mathrm{t}^2}{2}-\frac{t^3}{3\cdot\tau_B}\right\},\quad 0\le t\le\tau_B. \tag{35}$$

Ratios for $|T|_{max}$ braking the crane to a complete stop have a form similar to (23), (24), but with replacement in them $\tau_s\to\tau_B, P\to P_B$:

$$|T|_{\max}=|T|\big|_{t=\tau_B}=c\cdot\frac{(P_B-W)}{6\cdot m_1}\cdot\tau_B^2=\frac{\mathrm{m}_2\cdot g}{H}\cdot\frac{(P_B-W)}{6\cdot m_1}\cdot\tau_B^2. \tag{36}$$

Relationship between $|T|$ and t in the braking process of the crane mechanism has a form similar to the one shown in Fig. 2 with an obvious substitution $|T|_{max}$ (36) and on the time axis (t) $\tau_s\to\tau_B$.

Let us further elucidate the laws of motion of masses: $m_2-x_2(t)$ and $m_1-x_1(t)$, at which the motion quality criterion (31) under initial conditions is fulfilled:

$$x_1\big|_{t=0}=x_2\big|_{t=0}=0;\quad \dot{x}_1\big|_{t=0}=\dot{x}_2\big|_{t=0}=V, \tag{37}$$

where: V – initial speed of uniform motion (without pendulum oscillations) of the load lifting mechanism (load trolley and load on the rope). For $x_2(t)$ one has:

$$\mathrm{x}_2(\mathrm{t})=\frac{c}{m_1\cdot m_2}\cdot(P_B-W)\cdot\left\{\frac{\mathrm{t}^4}{24}-\frac{t^5}{60\cdot\tau_B}\right\}+V\cdot t, \tag{38}$$

and for $x_1(t)$ it is easy to determine the following law of motion:

$$\mathrm{x}_1(\mathrm{t})=\frac{c}{m_1\cdot m_2}\cdot(P_B-W)\cdot\left\{\frac{\mathrm{t}^4}{24}-\frac{t^5}{60\cdot\tau_B}\right\}+V\cdot t+\frac{(P_B-W)}{m_1}\cdot\left\{\frac{\mathrm{t}^2}{2}-\frac{t^3}{3\cdot\tau_B}\right\}, \tag{39}$$

Formulas (38) and (39) take place only in the time interval $t\in[0,\tau_B]$.

The estimation (30) is also valid for the case of braking of the hoisting mechanism:

$$\tau_B<\sqrt{\frac{12\cdot H}{g\cdot\left(1+\frac{m_2}{m_1}\right)}}. \tag{40}$$

It should be noted that the control of the motion parameters of the trolley and the load on the rope, as well as the actual values of the τ_s,τ_B belong to a special mechatronic control system [7].

2. Calculation of optimal modes of motion of the lifting mechanism of the bridge/bridge crane based on D.L. Landau's model [1].

2.1 Start-up mode. We use the approach of [6] to analyze the dynamics and optimize the modes of motion of the elements of the hoisting mechanism of an overhead/bridge crane. In this case, the model of the problem is presented in Fig. 3 and is reduced to the following.

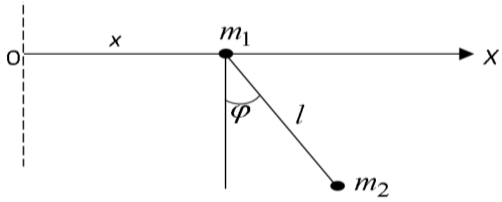

Fig. 3. Geometry of the problem (L.D. Landau model [1])

Flat pendulum with mass m_2, whose point of fixture (with point mass m_1) can move along a horizontal straight line (Fig. 3).

Entering the coordinate of x point and m_1 (crane load trolley) and angle φ between the pendulum cable (of length l) and the vertical axis. The load has mass m_2 and its coordinate OX :

$$x_2 = x + l \cdot sin\varphi. \tag{41}$$

For $\varphi << 1$ instead of (41) one has:

$$x_2 = x + l \cdot \varphi. \tag{42}$$

This formulation of the problem is fundamentally different from the model of N.A. Lobov considered above, since in the model of L.D. Landau x_2 is a coordinate depending on x ($x \equiv x_1$ since N.A. Lobov's model). In addition, in the L.D. Landau model in the refined formulation one should also consider the motion in the direction perpendicular to the axis *OX*. (Recall that in the model of N.A. Lobov x_2 – independent from x_1 coordinate, and the motion of the system is considered only in the horizontal direction). Therefore, the results obtained in the L.D. Landau model will differ from those obtained in the N.A. Lobov model. The authors of this study, believe that the L.D. Landau model, refined in this study and supplemented by taking into account the forces acting on mass m_1 is more accurate.

The Legrange function for this problem has the form:

$$L = \frac{(m_1 + m_2)}{2} \cdot \dot{x}^2 + \frac{m_2}{2} \cdot l^2 \cdot \dot{\varphi}^2 + m_2 \cdot l \cdot \dot{x} \cdot \dot{\varphi} \cdot cos\varphi + m_2 gl \cdot cos\varphi + x \cdot F(t), \tag{43}$$

where: $F(t)$ describes the reciprocal of all forces (driving/braking, frictional forces) acting on the mass m_1 in the horizontal direction (along the axis *OX*)and create its motion along the specified axis. Further, to analyze the transient modes of crane operation (starting, braking), we consider the force to be $F(t)$ constant value and not depend on time (t).

Then the equation of motion of the mechanical system at such a Lagrange function L (43) are as follows:

$$\begin{cases} (m_1 + m_2) \cdot \ddot{x} + m_2 \cdot l \cdot \left(\ddot{\varphi} cos\varphi - sin\varphi \cdot \dot{\varphi}^2\right) = F; \\ g \cdot sin\varphi + l \cdot \ddot{\varphi} + \ddot{x} cos\varphi = 0. \end{cases} \tag{44}$$

For $\varphi << 1$ let us linearize the equations of the system (44) $(sin\varphi \approx \varphi; \quad cos\varphi \approx 1)$:

$$\begin{cases} (m_1 + m_2) \cdot \ddot{x} + m_2 \cdot l \cdot \ddot{\varphi} = F; \\ g \cdot \varphi + l \cdot \ddot{\varphi} + \ddot{x} = 0. \end{cases} \tag{45}$$

After simple transformations, from (45) we can obtain:

$$(m_1 + m_2) \cdot g \cdot \varphi + m_1 \cdot l \cdot \ddot{\varphi} = -F, \tag{46}$$

or:

$$\frac{(m_1+m_2)\cdot g\cdot\varphi}{m_1\cdot l}+\ddot{\varphi}=\frac{-F}{m_1\cdot l}. \qquad (47)$$

Note the notation:

$$\Omega^2=\frac{g\cdot(m_1+m_2)}{m_1\cdot l}, \qquad (48)$$

where: Ω – is the natural frequency of oscillations of the system under consideration. From (47), (48) it is easy to obtain:

$$\varphi=\frac{\left\{\ddot{\varphi}+\frac{F}{m_1\cdot l}\right\}\cdot(-1)}{\Omega^2}. \qquad (49)$$

One shall examine the laws of motion $\varphi(t)$ with this mechanical system (within the model's parameters, that is being considered, and the approximation $\varphi<<1$), that lays out the nest motion quality criteria (at the start, for the duration τ_Π):

$$I^*=\left\{\frac{1}{\tau_\Pi}\cdot\int_0^{\tau_s}\{\varphi\}^2\,\mathrm{dt}\right\}^{\frac{1}{2}}\Rightarrow\min. \qquad (50)$$

Given (49), the criterion (50) can be represented in this form:

$$I^*=\left\{\frac{1}{\tau_\Pi}\cdot\int_0^{\tau_s}\left[\frac{\ddot{\varphi}+\frac{F}{m_1\cdot l}}{\Omega^2}\right]^2\mathrm{dt}\right\}^{\frac{1}{2}}\Rightarrow\min. \qquad (51)$$

A necessary condition for the realization of criterion (50) or (51) is the Euler-Poisson equation:

$$\varphi^{(IV)}=0, \qquad (52)$$

with condition that $F=const$.

We will search for the solution of equation (52) in the form of cubic spline by t:

$$\varphi(t)=b_0+b_1\cdot t+b_2\cdot t^2+b_3\cdot t^3. \qquad (53)$$

Constants (b_0,b_1,b_2,b_3) for the crane starting mode is found from the following terminal conditions:

$$\varphi|_{t=0}=0;\quad \dot{\varphi}|_{t=0}=0;\quad \ddot{\varphi}|_{t=0}=-\frac{F}{m_1\cdot l};\quad \dot{\varphi}|_{t=\tau_s}=-\frac{V_0}{l}. \qquad (54)$$

Therefore:

$$b_0=0;\quad b_1=0;\quad b_2=-\frac{F}{2\cdot m_1\cdot l};\quad b_3=\left\{\frac{\left(-\frac{V_0}{l}\right)+\frac{F}{m_1\cdot l}\cdot\tau_s}{3\cdot\tau_s^2}\right\}. \qquad (55)$$

In (54), (55) V_0 – speed of steady motion of the crane trolley at the end of start-up.

Finally, $\varphi(t)$ assumes the following form:

$$\varphi(t)=\left(-\frac{\mathrm{F}}{2\mathrm{m}_1\cdot l}\right)\cdot t^2+\left(\frac{F\cdot\tau_s}{\mathrm{m}_1\cdot l}-\frac{\mathrm{V}_0}{l}\right)\cdot\frac{1}{3\cdot\tau_s^2}\cdot t^3. \qquad (56)$$

in (56) $t\in[0,\tau_s]$.

The second equation of the system (45) gives:

$$\ddot{x} = -\{g\varphi + l\ddot{\varphi}\} = -\left\{ g \cdot \left[\left(-\frac{F}{2m_1 \cdot l} \right) \cdot t^2 + \left(\frac{F \cdot \tau_s}{m_1 \cdot l} - \frac{V_0}{l} \right) \cdot \frac{t^3}{3 \cdot \tau_s^2} \right] \right\} - \\ - \left\{ \frac{-F}{m_1} + \left(\frac{F \cdot \tau_s}{m_1} - V_0 \right) \cdot \frac{2t}{\tau_s^2} \right\}. \quad (57)$$

Twice integrating (57) at t with zero initial conditions $\left(x|_{t=0} = \dot{x}|_{t=0} = 0 \right)$, for $x(t)$:

$$x(t) = \frac{F \cdot t^2}{2m_1} - \left(\frac{F \cdot \tau_s}{m_1} - V_0 \right) \cdot \frac{t^3}{3 \cdot \tau_s^2} + \frac{F \cdot g \cdot t^4}{24 \cdot m_1 \cdot l} - g \cdot \left(\frac{F \cdot \tau_s}{m_1 \cdot l} - \frac{V_0}{l} \right) \cdot \frac{t^5}{60 \cdot \tau_s^2}. \quad (58)$$

For $x_2(t) = x(t) + l \cdot \varphi(t)$:

$$x_2(t) = \frac{F \cdot g \cdot t^4}{24 m_1 \cdot l} - g \cdot \left(\frac{F \cdot \tau_s}{m_1 \cdot l} - \frac{V_0}{l} \right) \cdot \frac{t^5}{60 \cdot \tau_s^2}. \quad (59)$$

in (58), (59) $t \in [0, \tau_s]$.

The maximum load of the cable, as well as its current load $F_H(t)$, can be found from the relationship:

$$F_H(t) = \frac{m_2 \cdot g}{l} \cdot |x_2(t) - x_1(t)| = \frac{m_2 \cdot g}{l} \cdot |\varphi(t)| \cdot l = m_2 \cdot g \cdot |\varphi(t)| = \\ = m_2 \cdot g \cdot \left| \left(-\frac{F}{2m_1 \cdot l} \right) \cdot t^2 + \left(\frac{F \cdot \tau_\Pi}{m_1 \cdot l} - \frac{V_0}{l} \right) \cdot \frac{t^3}{3 \cdot \tau_\Pi^2} \right|. \quad (60)$$

$$[F_H(t)]_{max} = F_H(t^*), \quad t^* = \left[\frac{F \cdot \tau_s^2}{(m_1 \cdot l)} \right] \cdot \left(\frac{F \cdot \tau_s}{m_1 \cdot l} - \frac{V_0}{l} \right)^{-1}. \quad (61)$$

2.2. *Braking mode.* In the crane system-braking mode, the considerations given in paragraph 2.1 above are repeated. Only replacements should be made: $\tau_s \to \tau_B$; $F_s = F \to F_B$ and accept the following terminal conditions for the definition $\varphi(t)$:

$$\varphi|_{t=0} = 0; \quad \dot{\varphi}|_{t=0} = -\frac{V_0^*}{l}; \quad \ddot{\varphi}|_{t=0} = -\frac{F_B}{m_1 \cdot l}; \quad \dot{\varphi}|_{t=\tau_B} = 0. \quad (62)$$

In (62) V_0^* – initial speed of uniform motion of the crane trolley, which decreases to zero at the moment of the end of the braking process, i.e. at $t = \tau_B$.

Coefficient $\left(\tilde{b}_0, \tilde{b}_1, \tilde{b}_2, \tilde{b}_3 \right)$ for denoting $\varphi(t)$ when braking, therefore:

$$\varphi(t) = \tilde{b}_0 + \tilde{b}_1 \cdot t + \tilde{b}_2 \cdot t^2 + \tilde{b}_3 \cdot t^3, \quad (63)$$

Accounting for (62) assumes the following form:

$$\begin{cases} \tilde{b}_0 = 0; \ \tilde{b}_1 = -\frac{V_0^*}{l}; \ \tilde{b}_2 = -\frac{F_B}{2m_1 \cdot l}; \\ \tilde{b}_3 = \frac{1}{3 \cdot \tau_B^2} \cdot \left\{ \frac{V_0^*}{l} + \frac{F_B \cdot \tau_B}{m_1 \cdot l} \right\}. \end{cases} \quad (64)$$

Therefore, the expression $\varphi(t)$ for braking assumes the following form:

$$\varphi(t) = \left(-\frac{V_0^*}{l} \right) \cdot t + \left(-\frac{F_B}{2m_1 \cdot l} \right) \cdot t^2 + \frac{1}{3 \cdot \tau_B^2} \cdot \left\{ \frac{V_0^*}{l} + \frac{F_B \cdot \tau_B}{m_1 \cdot l} \right\} \cdot t^3 \quad (65)$$

For $\ddot{x}(t)$ from the expression (45):

$$\ddot{x} = -(g\cdot\varphi + l\cdot\ddot{\varphi}) = (-g)\cdot\left(-\frac{V_0^*}{l}\right)\cdot t + \left(+\frac{F_B\cdot g}{2m_1\cdot l}\right)\cdot t^2 + \frac{(-g)}{3\cdot\tau_B^2}\cdot\left(\frac{V_0^*}{l} + \frac{F_B\cdot\tau_B}{m_1\cdot l}\right)\cdot t^3 + \\ + \frac{F_B}{m_1} - \frac{2t}{\tau_B^2}\cdot\left\{V_0^* + \frac{F_B\cdot\tau_B}{m_1}\right\}. \quad (66)$$

Integrating twice over t (66) with the following initial conditions:

$$x\big|_{t=0} = 0;\quad \dot{x}\big|_{t=0} = V_0^*. \quad (67)$$

Then:

$$x(t) = \left(+\frac{gV_0^*}{6l}\right)\cdot t^3 + \frac{F_B\cdot g\cdot t^4}{24\cdot m_1\cdot l} + \frac{(-g)}{60\cdot\tau_B^2}\cdot\left(\frac{V_0^*}{l} + \frac{F_B\cdot\tau_B}{m_1\cdot l}\right)\cdot t^5 + \frac{F_B\cdot t^2}{2\cdot m_1} + \\ + V_0^*\cdot t - \frac{t^3}{3\cdot\tau_B^2}\cdot\left\{V_0^* + \frac{F_B\cdot\tau_B}{m_1}\right\}. \quad (68)$$

$x_2(t) = x(t) + l\cdot\varphi(t)$ come from the relation:

$$x_2(t) = \left(+\frac{gV_0^*}{6l}\right)\cdot t^3 + \frac{F_B\cdot g\cdot t^4}{24m_1\cdot l} + \frac{(-g)}{60\cdot\tau_B^2}\cdot\left(\frac{V_0^*}{l} + \frac{F_B\cdot\tau_B}{m_1\cdot l}\right)\cdot t^5. \quad (69)$$

The rope load as well as its maximum value for the braking mode of the crane system can be found from the relations:

$$F_H(t) = \frac{m_2\cdot g}{l}\cdot|x_2(t) - x_1(t)| = m_2\cdot g\cdot|\varphi(t)| = m_2\cdot g\cdot|\varphi(t)| = \\ = m_2\cdot g\cdot\left|\left(-\frac{V_0^*}{l}\right)\cdot t - \frac{F_B\cdot t^2}{2m_1\cdot l} + \frac{1}{3\cdot\tau_B^2}\cdot\left(\frac{V_0^*}{l} + \frac{F_B\cdot\tau_B}{m_1\cdot l}\right)\cdot t^3\right|. \quad (70)$$

$$[F_H(t)]_{max} = F_H(t^{**}),\quad t^{**}\in[0,\tau_B], \quad (71)$$

where t^{**} is found as a real positive root of the quadratic equation:

$$\frac{1}{\tau_B^2}\cdot\left(\frac{V_0^*}{l} + \frac{F_B\cdot\tau_B}{m_1\cdot l}\right)\cdot t^2 - \frac{F_B\cdot t}{m_1\cdot l} - \frac{V_0^*}{l} = 0. \quad (72)$$

It should be noted that both in the operating mode of the crane system at its start-up and at its deceleration $\varphi(t)\neq 0$ with $t > \tau_s$ else $t > \tau_B$, i.e. there are residual vibrations of the load on the rope.

When approaching $t\to\tau_s$ else $t\to\tau_B$ one (for $\varphi(t)$ else $\dot{\varphi}(t)$)will be non-zero and therefore, due to the inertial properties of the system, residual oscillations will exist.

The results obtained in this study allow only to determine such modes of crane system startup/braking, at which during the transition period ($t\in[0,\tau_s]$ else $t\in[0,\tau_B]$) oscillations of the load on the rope are minimized. Actually, such modes of operation of the hoisting mechanism of the bridge/bridge crane are called optimal in this paper.

Conclusion:

1. Dynamic analysis of transient modes of motion (startup/braking) of hoisting mechanisms of overhead/urban cranes has been carried out. The scientific novelty of the work consists in the fact that for the first time correct (terminal) initial and final conditions of motion of the crane system in the transient mode of its functioning (in the period of start-up/braking), in which there is (and in the conditions stated) a physical cause of such motion, and the laws of motion are constructed by means of spline functions (in time), and the defect of the time spline is equal to zero (that is, the number of terminal conditions corresponds to the number of uncertain constants of the time spline). The latter circumstance indicates the correctness of the law of motion of the crane system in time defined in this way.

2. Within the framework of classical models of motion of the crane system (Lobov N.A. [3] and Landau L.D. [1]) the basic parameters of stresses in ropes and kinematic characteristics, at which pendulum oscillations of a cargo are minimized exactly during the whole transient mode of functioning, are determined.

3. The results obtained in the work can be further used to clarify and improve the existing engineering methods for calculating the parameters of lifting mechanisms of bridge/grading type cranes in order to reduce the negative impact of pendulum oscillations arising in the system.

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АНАЛІЗ ДИНАМІКИ ТА ОПТИМІЗАЦІЯ ПЕРЕХІДНИХ РЕЖИМІВ РУХУ ВАНТАЖОПІДЙОМНИХ МЕХАНІЗМІВ МОСТОВИХ/МІСТОБУДІВЕЛЬНИХ КРАНІВ

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Анотація. Проведене дослідження режимів роботи вантажопідйомних механізмів і машин, зокрема, мостових та містобудівельних кранів. На основі динамічного аналізу режимів роботи механізмів вказаних машин встановлений характер виникнення навантажень у їх пружних елементах (канатах). Виконано синтез оптимальних перехідних режимів роботи (пуск/гальмування) містобудівельних та мостових кранів. Використані моделі Л.Д. Ландау та Н.А. Лобова для фізико-механічного аналізу руху вказаних машин, на основі котрих із залученням методів класичного варіаційного числення отримані аналітичним шляхом закони руху вантажу та вантажного візка подібних кранів, які оптимізують (мінімізують) неминуче виникаючі маятникові коливання вантажу на канаті (як у період пуску крана, так і при його гальмуванні, до повної зупинки).

Оцінка режимів руху у роботі здійснення шляхом використання критерію з підінтегральними функціями у вигляді «енергії» пришвидшень вищих порядків (третього, четвертого і т.д.). Саме такі критерії використовуються при виборі режимів руху вантажопідйомних механізмів кранів мостового/містобудівельного типу, які мають у своєму складі пружні елементи (канати). Вказаний підхід дає можливість досягнути неперервності функцій швидкостей і пришвидшень різних ланок системи, що і призводить до значного зменшення їх коливань. Аналіз режимів руху системи, яка описується основною координатою руху і пружними координатами, дозволив встановити закономірність використання запропонованого критерію для мінімізації пружних коливань окремих ланок (канатів) вантажопідйомного механізму крана.

Результати даного дослідження можуть бути у подальшому використані для уточнення й вдосконалення існуючих інженерних методів розрахунку навантажень у гнучких/пружних елементах (канатах) мостових та містобудівельних кранів як при проектуванні вантажопідйомних механізмів, так і у режимах їх реальної експлуатації при застосуванні у керуванні рухом вантажного візка мехатронних систем управління.

Ключові слова: аналіз, динаміка, оптимізація, перехідні режими руху, пуск, гальмування, вантажопідйомні механізми, мостові та містобудівельні крани.

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