UDC 624.012.4:624.046.2/.4 DOI: 10.31650/2786-6696-2025-11-33-42

PRACTICAL ANALYSIS METHOD OF REINFORCED CONCRETE FRAME STRUCTURES WITH CONSIDERATION OF CREEP AND CRACKING IN CONCRETE

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Abstract. The article is devoted to enhancing the practical method of reinforced concrete frame structure analysis with consideration of the creep and cracking processes in concrete. The proposed method is based on the usage of a linearized diagram of a bending moment and curvature relationship for reinforced concrete bending elements. The formulas for evaluation of the bending stiffness of reinforced concrete elements with consideration of creep and cracking of concrete are given. The proposed formulas are based on the linear creep theory that assumes a linear relationship between stresses and elastic deformations as well as between stresses and creep deformations. The algorithm for drawing of linearized diagrams "bending moment vs curvature" for reinforced concrete elements of rectangular cross-section with consideration of creep and cracking is described.

The analysis of portal reinforced concrete frames used in the experimental research of other authors was made to verify the proposed practical method. Experimental research provided tests of frames by the short-term loads for the definition of the bearing capacity and by the long-term loads of the different intensities. According to geometrical dimensions, reinforcement schemes, and materials properties of the experimental frames the linearized diagrams "bending moment vs curvature" were drawn with the usage of the proposed algorithm. Tables with the comparison of bending moments values obtained by the analysis results with values obtained during experimental research are given. Statistic analysis of the ratio of theoretical values of bending moments to experimental values at the action of short-term and long-term loads shows good agreement with differences less than 10%.

Keywords: reinforced concrete, creep, cracking, bending moment, curvature.

Introduction. Concrete, which is the main component of reinforced concrete structures, has physical and mechanical properties that change over time and under the influence of the applied load. Consideration of a concrete creep plays a particularly important role in determining and predicting the stress-strain state of building structures during the restoration, reconstruction and decommissioning of construction facilities. Thus, the development and improvement of practical methods for the analysis of reinforced concrete structures that are applicable for consideration of the processes of cracking and creep in concrete is an important task.

Analysis of Recent Research. The work of many scientists has been devoted to the study of the effect of concrete creep on the stress-strain state [1-12].

Modern works distinguish between the following concrete creep theories:

– The theory of aging.

– The theory of elastic inheritance.

– The theory of hereditary aging.

According to the aging theory, creep deformations depend on stress levels and time. It is assumed that the creep curves corresponding to the loading at different ages are parallel to each other.

The theory of elastic inheritance assumes complete reversibility of creep deformations. In addition, according to this theory, the elastic modulus of concrete has a constant value and does not depend on the age of the concrete. Therefore, the theory of elastic inheritance can be used only for the analysis of structures made of "old" concrete.

The theory of hereditary aging is a combination of the above-mentioned aging theory and elastic inheritance theory, which allows us to characterize the creep processes of concrete most completely.

The contemporary normative documents [13, 14] regulate the determination of only the maximum creep rate at a constant stress level. In this case, it is assumed that creep depends on the duration of the load, concrete class, open surface modulus, and cement class.

Purpose of Research. The purpose of this work is to study the effect of creep and cracking in concrete on the bending moment-curvature diagrams and to improve the analysis method of reinforced concrete frame structures using bending moment-curvature diagrams that allow considering the effect of creep and cracking in concrete.

Research Method. The linear creep theory based on linear relationships between both stresses and elastic strains and between stresses and creep strains is adopted in this paper to consider the influence of time on the physical and mechanical properties of concrete.

In analysis of reinforced concrete structures with creep and cracking consideration, the stiffness of their elements, and thus the behavior of the structure under load, depends on the distribution of internal forces and their changes over time ($t–t_0$).

In this paper, it is assumed:

t – is the current age of concrete for which the stress-strain state of the structure is determined;

t_0 – is the age of concrete at the time of load application.

The time starts from the moment the concrete begins to harden. The term "age of concrete" means the time interval from the beginning of concrete hardening to the considered time (t or t_0).

The stiffness of reinforced concrete structural elements in the case of bending under the proposals outlined in [2, 3], can be determined by the following formulas:

– in the absence of cracks in the concrete:

$$B^*(t) = E_c(t)\, I_{red}\, \frac{1+\rho_f \rho_1 \alpha(t)\big(1+\varphi(t,t_0)\big)}{\big(1+\varphi(t,t_0)\big)\big[1+\rho_f \alpha(t)\big(1+\varphi(t,t_0)\big)\big]}; \tag{1}$$

where $B^*(t)$ is long-term bending stiffness with the creep consideration of bent reinforced concrete element under the action of the long-term load at the moment of time t;

$E_c(t)$ is the elasticity modulus of concrete at the moment of time t;

I_{red} is the reduced moment of inertia of the reinforced concrete element cross-section;

ρ_f is a reinforcement ratio, equal to the ratio of the reinforcement area to the area of concrete, but no more than 0.02;

ρ_1 is the factor that characterizes the geometry of the cross-section, position of reinforcements and stress state;

$\alpha(t)$ is the ratio of the reinforcement elasticity modulus to the elasticity modulus of concrete;

$\varphi(t,t_0)$ is creep coefficient;

– in case of cracks in the concrete:

$$B_L^*(t) = \frac{B^*(t)}{1+\varphi(t,t_0)\left(\beta_1+\beta_2\left(\frac{M_{crc}}{M_L}\right)^2\right)}, \tag{2}$$

where $B_L^*(t)$ is long-term bending stiffness with the creep and cracking consideration of bent reinforced concrete element under the action of the long-term load at the moment of time t;

β_1 and β_2 are factors that depend on cross-section parameters;

M_{crc} is a value of the bending moment that causes the first crack;

M_L is the bending moment caused by long-term loads.

The following algorithm for drawing bending moment-curvature diagrams with consideration of the creep and cracking processes is proposed in this article:

1. The following parameters are used as initial: class of concrete C; amount of air involved v; the specific weight of cement dough in the concrete mixture P_T; relative humidity of the environment W; the design value of reinforcement compressive strength R_{sk} for the first group of ultimate states; the elasticity modulus of reinforcement E_s; height h and width b of the cross-section; constant values $c_k = 0.5$ and $k = 1$ applied according to the recommendation [15]; a parameter f_0 that characterizes the curvature of the stress curve in the compressed zone of concrete.
2. The open surface modulus M_0 is calculated according to the given shape and dimensions of the cross-section by the following formula:

$$M_0 = \frac{u}{A}, \tag{3}$$

where u is the perimeter of the cross-section that corresponds with the open surface;
A is a cross-sectional area.

3. According to the applied initial parameters, the table parameters are evaluated from the reference tables given in the recommendation [15] coefficients v_k and v_c that depends on the class of concrete in terms of compressive strength; dimensionless coefficient k_c for concretes with coarse aggregate and for fine-grained concretes with quartz sand; parameter s that characterize the influence of elastic properties of aggregate in concrete; coefficient ξ_{3c} that depends on the humidity of the environment; coefficients γ, γ_1, and ξ_{2c} that depend on the modulus of the open surface; coefficient d that depends on the time of sample loading and the modulus of the open surface.
4. The physical and mechanical properties of concrete at the age of t and t_0 are calculated:
   - the cubic strength of concrete $f^G_{ck,cube}(t)$ that is determined in accordance with the recommendations [15] and is based on the strength corresponding to a given class of concrete at the age of t = 28 days by the following formula:

$$f^G_{ck,cube}(t) = \left(1 + \frac{23}{55 + C} \cdot \frac{t - 28}{t + 11}\right) C, \tag{4}$$

   - the initial elasticity modulus of concrete by formula:

$$E_c(t) = \frac{400 s f^G_{ck,cube}(t)}{s P_T + f^G_{ck,cube}(t)}. \tag{5}$$

5. The following geometric properties of the cross-section are determined: the ratio $\alpha(t)$ of the concrete elasticity modulus $E_c(t)$ to the reinforcement elasticity modulus E_s; the cross-sectional area $A_{red}(t)$ of the reduced section; the first moment of the reduced section $S_{red}(t)$ concerning the axis passing through the lower edge of the section; coordinate of the gravity center of the reduced section $y_{red}(t)$ concerning the axis passing through the lower edge of the section; moment of inertia of the concrete section concerning its central axes I_{c0}; moment of inertia of concrete section $I_c(t)$ concerning the central axis of the reduced cross-section; moments of inertia of the reinforcement in the stretched $I_s(t)$ and compressed $I_s'(t)$ zones; moment of inertia of the reduced section $I_{red}(t)$ concerning its central axis; coefficient that characterizes the geometry of the cross-section of the element, the location of the reinforcement, and the stress state $\rho_1(t)$; reinforcement ratio of the cross-section ρ_f.
6. The creep rate of concrete at age t under load applied at age t_0 is calculated by the formula:

$$C(t,t_0) = \frac{1}{E_c(t_0)} - \frac{1}{E_c(t)} + C^N(\infty,28)\xi_{2c}\xi_{3c}\Omega(t_0) f(t - t_0), \tag{6}$$

where $C^N(\infty,28)$ – is the ultimate value of the creep rate of concrete loaded at the age of 28 days after curing in natural conditions, calculated by the formula:

$$C^N(\infty,28) = k_c \left[\frac{(W + \nu)}{(C + 4.0)}\right]; \tag{7}$$

$\Omega(t_0)$ – is a function that considers the influence of the concrete aging on the creep rate, evaluated by the formula:

$$\Omega(t_0) = c_k + d\exp(-\gamma t_0); \tag{8}$$

$f(t - t_0)$ – is a function that considers increasing the creep rate with time, evaluated by the formula:

$$f(t - t_0) = 1 - k\exp\left[-\gamma_1 (t - t_0)\right]. \tag{9}$$

7. The bending moment at which cracking processes are beginning in the reinforced concrete element at age *t* is calculated:

$$M_{crc}(t) = W_{pl}(t)\, f_{ctk}(t), \tag{10}$$

where $W_{pl}(t)$ – is the elastic-plastic section modulus;

f_{ctk} – is the characteristic value of the concrete strength in tension.

8. The ultimate value of bending moment that can withstand the cross-section of a reinforced concrete element at age *t* is calculated:

$$M_u(t) = \frac{f_{ck,prism}(t)\, b x_{\min}^2(t, t_0)}{2 + f_0} + R_{sk} A_{sc}\left[h - c - x_{\min}(t, t_0)\right], \tag{11}$$

where $f_{ck,prism}(t)$ – is the characteristic value of the prismatic compression strength of concrete at the age *t* that can be defined by the formula (4) with the usage of the characteristic value of the prismatic compression strength of concrete $f_{ck,prism}$ at age $t = 28$ days instead of concrete class *C*;

$x_{min}(t,t_0)$ – is the minimum height of the compressed zone of concrete;

c – is the concrete cover of reinforcement in the tension zone.

9. The long-term bending stiffness with creep consideration $B^*(t)$ of a reinforced concrete element is calculated by formula (1).

10. The long-term bending stiffnesses with creep and cracking consideration $B_L^*(t)$ of a reinforced concrete element are calculated by formula (2). In this case, the stiffnesses at values of long-term bending moments $M_L = M_{crc}(t)$ and $M_L = M_u(t)$ are calculated.

11. The ultimate values of the element curvature are calculated using the following formulas:

$$\chi_1 = \frac{M_{crc}}{B^*(t)}, \quad \chi_2 = \frac{M_{crc}}{B_L^*(t)}, \quad \chi_3 = \frac{M_u(t)}{B_L^*(t)}. \tag{12}$$

12. Based on the obtained values of the ultimate bending moments $M_{crc}(t)$, $M_u(t)$ and curvatures χ_1, χ_2, χ_3, a bending moment-curvature diagram is drawn.

Results of Research. To verify the proposed practical method for consideration of creep and cracking processes in concrete, the authors compared the analysis results with the experimental data obtained at the Kyiv National University of Civil Engineering and Architecture by A.Y. Barashikov, L.A. Murashko, and G.M. Reminets [5]. These experimental studies contain information about the testing of U-shaped reinforced concrete frames, the design scheme of which is shown in Fig. 1. There were the following series of frames: RP_{28} tested by a single short-term load to failure; R0.25 and R0.5 tested by the long-term load of different intensity.

The frames had the following design dimensions in the axes: height $H = 1125$ mm; span length $L = 1500$ mm. The beam had a cross-section of 120 × 170 mm, and the column had a cross-section of 120 × 135 mm. Class A-II reinforcement with yield strength $\sigma_y = 338$ MPa and elasticity modulus $E_s = 213000$ MPa was used to reinforce the frame. The columns are reinforced by two rods with a diameter of 10 mm near the inner face and two rods with a diameter of 14 mm near the outer face of the cross-section. The beam is reinforced by two rods with a diameter of 14 mm near the bottom and top faces of the cross-section. In this case, the top reinforcement was located only above the supports on a section of 500 mm in length.

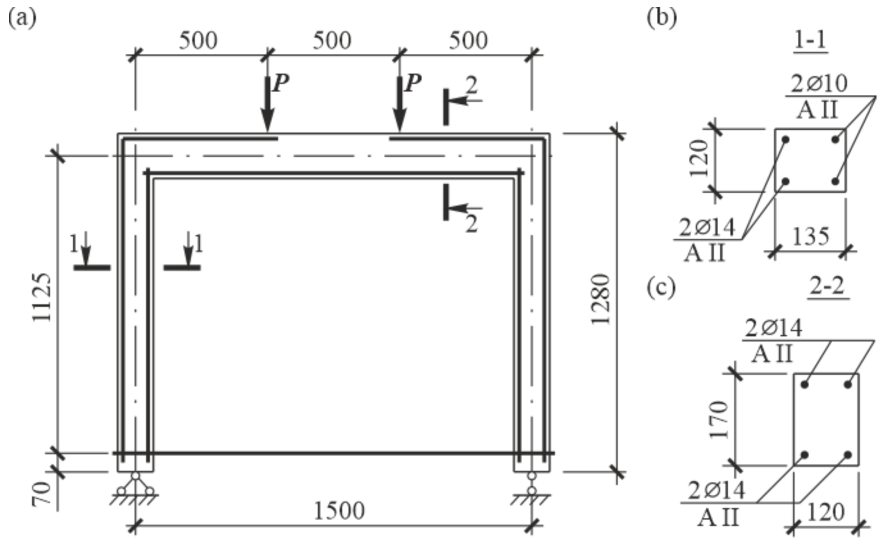

Fig. 1. Design scheme of frames RP_{28}, R0.25 and R0.5:
a – scheme of reinforcements; b – columns cross-section; c – beam cross-section

Samples of the frame RP_{28} were tested under a single short-term load until failure at the age of 28 days. The prismatic strength of concrete at the time of testing was f_{cd} = 36.2 MPa. The test samples were loaded with two concentrated forces applied at a distance of 500 mm from the columns axes. The load was applied to the frame in steps of 5 kN.

Samples of frames R0.25 and R0.5 were used to study the effect of long-term action of load respectively of 10 and 20 kN, applied at the age of 28 days. The duration of the load was up to 293 days.

For the opening of the static indeterminacy of the experimental frames, the force in the tightening was measured using a flat dynamometer. Based on the experimental values of the force in the tightening H, the values of the support M_{sup} and span M_{sp} bending moments were calculated.

The bending moment-curvature diagrams drawn following the algorithm described above were used for the analysis of the experimental frames. The corresponding diagrams for the designed cross-sections of the columns and beam are shown in Figs. 2 and 3.

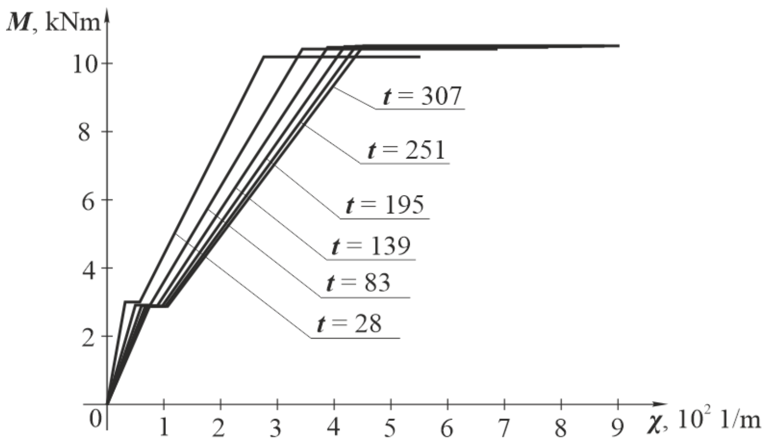

Fig. 2. Bending moment vs. curvature diagram for the column elements

Using a program compiled in the computer mathematics system MATLAB, according to the algorithm for determining the stress-strain state of reinforced concrete frame structures proposed in [5], the above-described reinforced concrete frame RP_{28} was calculated at each load stage. Table 1 shows a comparison of the experimental values of bending moments with the theoretical ones obtained by analysis with the usage of the bending moment-curvature diagrams drawn according to the proposed methodology. The table also shows a statistical assessment of the distribution of the ratio of the experimental and theoretical values of bending moments M_{exp}/M_{teor}.

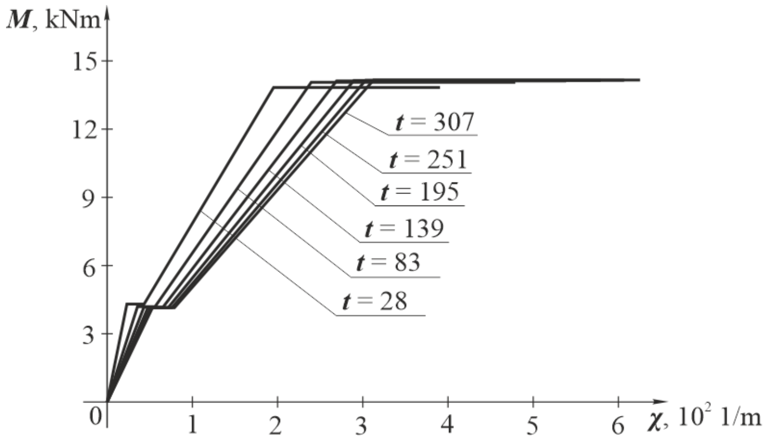

Fig. 3. Bending moment vs. curvature diagram for beam elements

Table 1 – Comparison of experimental and theoretical values of bending moments in the frame series RP_{28} [5]

P, kN	H^{exp}, kN	M_{sup}^{exp}, kN m	M_{sup}^{teor}, kN m	$\frac{M_{sup}^{exp}}{M_{sup}^{teor}}$	M_{sp}^{exp}, kN m	M_{sp}^{teor}, kN m	$\frac{M_{sp}^{exp}}{M_{sp}^{teor}}$
2	0.298	0.336	0.3513	0.9556	0.6643	0.6487	1.0240
4	0.476	0.533	0.6715	0.7937	1.467	1.3285	1.1043
6	0.796	0.892	1.0072	0.8856	2.108	1.9928	1.0578
8	1.200	1.344	1.343	1.0007	2.656	2.657	0.9996
10	1.961	2.196	1.6787	1.3082	2.804	3.3213	0.8442
15	2.678	2.999	3.1516	0.9516	4.501	4.3484	1.0351
20	3.352	3.754	3.9478	0.9509	6.246	6.0522	1.0320
25	4.360	4.883	4.7715	1.0234	7.617	7.7285	0.9856
30	5.670	6.350	5.5655	1.1410	8.65	9.4345	0.9168
35	6.010	6.731	6.4334	1.0463	10.769	11.0666	0.9731
40	7.118	7.972	7.2438	1.1005	12.028	12.7562	0.9429
45	8.000	8.960	8.6582	1.0349	13.54	13.8418	0.9782
50	9.860	11.043			13.957		
Sample mean, M_x				1.016			0.991
Sample variance, D_x				0.017			0.005
Sample coefficient of variation, C_v				0.129			0.069
Confidence interval (at P = 0.95)		upper limit		1.087			1.028
		lower limit		0.945			0.954

Also, using the compiled program, the analysis of the experimental frames R0.25 and R0.5 was performed at load duration of up to 293 days. For frames R0.25, the analysis was performed at a constant load of 10 kN, and for frames R0.5 – 20 kN. Tables 2 and 3 show a comparison of the results of the experimental research and theoretical values of bending moments obtained by the proposed methodology, as well as a statistical assessment of the distribution of the ratio of theoretical and experimental values of bending moments M_{exp}/M_{teor}.

Table 2 – Comparison of experimental and theoretical values of bending moments in the frame series R0.25 under the long-term load of P = 10 kN [5]

t, days	H^{exp}, kN	M_{sup}^{exp}, kN m	M_{sup}^{teor}, kN m	$\frac{M_{sup}^{exp}}{M_{sup}^{teor}}$	M_{sp}^{exp}, kN m	M_{sp}^{teor}, kN m	$\frac{M_{sp}^{exp}}{M_{sp}^{teor}}$
0	1.74	1.95	1.6787	1.162	3.23	3.3212	0.973
13	1.72	1.93	1.6665	1.158	3.23	3.3335	0.969
27	1.65	1.85	1.6585	1.115	3.28	3.3415	0.982
41	1.63	1.83	1.6532	1.107	3.27	3.3468	0.977
55	1.63	1.83	1.6494	1.109	3.27	3.3506	0.976
69	1.63	1.83	1.6466	1.111	3.27	3.3534	0.975
83	1.62	1.81	1.6446	1.101	3.28	3.3554	0.978
97	1.62	1.81	1.6430	1.102	3.29	3.3570	0.980
111	1.62	1.81	1.6418	1.102	3.28	3.3582	0.977
125	1.62	1.81	1.6409	1.103	3.28	3.3591	0.976
139	1.62	1.81	1.6402	1.104	3.28	3.3598	0.976
153	1.61	1.80	1.6397	1.098	3.27	3.3603	0.973
167	1.61	1.80	1.6393	1.098	3.27	3.3607	0.973
181	1.61	1.80	1.6390	1.098	3.26	3.3610	0.970
195	1.60	1.79	1.6388	1.092	3.26	3.3612	0.970
209	1.60	1.79	1.6387	1.092	3.26	3.3613	0.970
223	1.60	1.79	1.6386	1.092	3.24	3.3614	0.964
237	1.59	1.78	1.6386	1.086	3.24	3.3614	0.964
251	1.59	1.78	1.6386	1.086	3.24	3.3614	0.964
265	1.59	1.78	1.6386	1.086	3.24	3.3614	0.964
279	1.59	1.78	1.6387	1.086	3.24	3.3613	0.964
293	1.59	1.78	1.6388	1.086	3.24	3.3612	0.964
Sample mean, M_x				1.103			0.972
Sample variance, D_x				0.000409			0.000034
Sample coefficient of variation, C_v				0.0183			0.0060
Confidence interval		upper limit		1.112			0.974
(at P = 0.95)		lower limit		1.095			0.969

Conclusions:

1. A practical model of reinforced concrete elements that considers the creep and cracking processes in concrete is proposed.
2. An algorithm for drawing linearized bending moment-curvature diagrams considering creep and cracking processes in concrete is formed.
3. A MATLAB program for drawing linearized bending moment-curvature diagrams considering creep and cracking processes in concrete has been developed.
4. A program in the computer mathematics system MATLAB for the analysis of reinforced concrete frame structures considering concrete cracking has been improved for concrete creep consideration.
5. Results of analysis made by the proposed methodology coincide with the results of experimental research of other authors, with difference of up to 10%.

Table 3 – Comparison of experimental and theoretical values of bending moments in the frame series R0.5 under the long-term load of P = 20 kN [5]

t, days	H^{exp}, kN	M_{sup}^{exp}, kN m	M_{sup}^{teor}, kN m	$\frac{M_{sup}^{exp}}{M_{sup}^{teor}}$	M_{sp}^{exp}, kN m	M_{sp}^{teor}, kN m	$\frac{M_{sp}^{exp}}{M_{sp}^{teor}}$
0	3.85	4.31	3.9478	1.092	6.02	6.0522	0.995
13	3.73	4.18	3.8903	1.074	6.11	6.1097	1.000
27	3.73	4.18	3.8471	1.087	5.87	6.1529	0.954
41	3.72	4.17	3.8156	1.093	5.85	6.1844	0.946
55	3.69	4.13	3.7919	1.089	5.85	6.2081	0.942
69	3.65	4.09	3.7734	1.084	5.87	6.2266	0.943
83	3.64	4.08	3.7589	1.085	5.88	6.2411	0.942
97	3.62	4.05	3.7474	1.081	5.9	6.2526	0.944
111	3.57	4.00	3.7381	1.070	5.82	6.2619	0.929
125	3.56	3.99	3.7305	1.070	5.8	6.2695	0.925
139	3.55	3.98	3.7243	1.069	5.81	6.2757	0.926
153	3.48	3.90	3.7193	1.049	5.87	6.2807	0.935
167	3.49	3.91	3.7151	1.052	5.86	6.2849	0.932
181	3.48	3.90	3.7116	1.051	5.85	6.2884	0.930
195	3.41	3.82	3.7087	1.030	5.92	6.2913	0.941
209	3.41	3.82	3.7063	1.031	5.92	6.2937	0.941
223	3.41	3.82	3.7043	1.031	5.92	6.2957	0.940
237	3.41	3.82	3.7027	1.032	5.92	6.2973	0.940
251	3.41	3.82	3.7013	1.032	5.92	6.2987	0.940
265	3.41	3.82	3.7001	1.032	5.92	6.2999	0.940
279	3.41	3.82	3.6992	1.033	5.92	6.3008	0.940
293	3.41	3.82	3.6984	1.033	5.92	6.3016	0.939
Sample mean, M_x				1.059			0.944
Sample variance, D_x				0.000592			0.0003
Sample coefficient of variation, C_v				0.0230			0.0197
Confidence interval (at P = 0.95)		upper limit		1.069			0.952
		lower limit		1.049			0.936

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ПРАКТИЧНИЙ МЕТОД РОЗРАХУНКУ ЗАЛІЗОБЕТОННИХ РАМНИХ КОНСТРУКЦІЙ З УРАХУВАННЯМ ПОВЗУЧОСТІ ТА ТРІЩИНОУТВОРЕННЯ В БЕТОНІ

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Анотація. Стаття присвячена вдосконаленню практичного метода розрахунку залізобетонних рамних конструкцій з урахуванням процесів повзучості та тріщиноутворення в бетоні. Запропонований метод заснований на використанні лінеаризованих діаграм взаємозв’язку між згинальним моментом та кривизною залізобетонного елементу, що згинається. Наведено формули для визначення згинальної жорсткості залізобетонних елементів з урахуванням повзучості бетону та тріщиноутворення в ньому. Запропоновані формули базуються на теорії лінійної повзучості, яка передбачає лінійну залежність як між напруженнями і пружними деформаціями, так і між напруженнями та деформаціями повзучості. Описано алгоритм побудови лінеаризованих діаграм «згинальний момент – кривизна» для залізобетонних елементів прямокутного поперечного перерізу з урахуванням повзучості та тріщиноутворення.

Для верифікації запропонованого практичного метода виконано розрахунки П-подібної залізобетонної рами, що використовувалась у експериментальних дослідженнях інших авторів. Експериментальні дослідження передбачали випробування рами короткочасним навантаженням для визначення несучої здатності та тривалим навантаженням різної інтенсивності. За допомогою наведеного алгоритму на підставі геометричних розмірів, схеми армування та властивостей матеріалів дослідних рам побудовані лінеаризовані діаграми «згинальний момент – кривизна» для елементів рамної конструкції, які використовувались в подальших розрахунках. Наведені таблиці порівняння значень згинальних моментів, отриманих за результатами розрахунків, із значеннями отриманими під час експериментальних досліджень. Статистичний аналіз відношення теоретичних значень згинальних моментів до експериментальних, як при короткочасній, так і при тривалій дії навантаження вказує на добрий збіг із різницею менше 10%.

Ключові слова: залізобетон, повзучість, тріщиноутворення, згинальний момент, кривизна.

Стаття надійшла до редакції 3.01.2025