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FEATURES OF MODELING REINFORCED CONCRETE PROTECTIVE STRUCTURES BY AN EXPLICIT METHOD IN CALCULATIONS FOR TEMPERATURE FORCE LOAD

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Abstract. Mathematical modeling is currently the basis for approximate methods of calculations and determination of the stress-strain state (SSS) of structures under temperature effects. It allows numerical finite element (FE) methods to obtain valid solutions to many complex problems in cases of force and temperature loads acting on statically indeterminate reinforced concrete structures, including taking into account plastic deformations and non-stationary three-dimensional temperature fields.

The article describes the main stages of explicit modeling of reinforced concrete protective structures under power loads and thermal problem features, based on the capabilities of the LS-DYNA software package. The algorithms of mathematical modeling with a detailed step-by-step justification of the applied dependencies of the explicit method are described. It is indicated that the correct choice of interaction criteria and substantiated models based on the analysis of the structure allows obtaining adequate results of the numerical experiment, confirmed by other researchers. Dependencies are given that allow calculating the function values at a future time step using already known function values at the current step and its derivatives. The calculation of the FE node speeds using the explicit method of integrating dynamic equations is performed using an expression that is an explicit numerical method for solving the dynamic equations. A basic expression is given for calculating the accelerations of FE nodes when performing approximation of time derivatives using the finite difference method.

For a complete set of FE, the fundamental possible displacements of nodes, the generalized equation of conservation of energy of a solid deformable body, which is discretely imposed on the FE mesh, are taken into account.

For the case of temperature loads in fire mode, an approach to solving a thermal problem is presented. It is shown that a substantiated methodology using elements of explicit and implicit methods allows one to correctly solve the posed thermal problem, taking into account the nonlinear deformation of the materials of the reinforced concrete protective structure and spatial temperature fields from the external temperature load.

Key words: explicit integration method, fire load, hexahedral finite element, reinforced concrete structures, thermal problem.

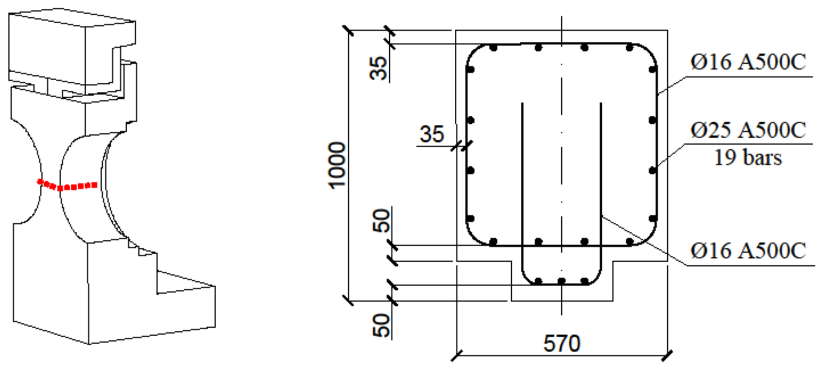
Introduction. The solution to the problem of determining the stress-strain state of reinforced concrete protective structures under the influence of force and temperature loads, variable in time (from fire and local heating by lasers, etc.) is impossible without the use of approximate numerical methods. At the present stage, two classes of them are used: explicit and implicit.

The main difference between them is how the new value of the function at the next point is calculated. Explicit methods are characterized by the calculation of new values of a function solely through the known values of the function and its derivatives at previous points, ease of implementation, which does not require solving systems of equations, and less stability for rigid systems, that is, systems with different time scales. Therefore, explicit methods may require a very small integration step to ensure accuracy. The main one of this group is the Euler method. Such methods are used to solve simple systems of differential equations.

Analysis of the latest research and publications. Recently, many software packages (LIRA, LS-DYNA) have successfully implemented implicit methods for solving problems with dynamic loads, including temperature loads from fire, seismic loads, etc. For this purpose, new finite elements [1–3] and algorithms implemented in domestic and foreign computer programs are being developed. In mathematical modeling of thermal problems using implicit methods, a new value of a function is calculated from an equation that includes both known values of the function at previous points and an unknown value at the next point. Such methods are much more complex to implement, since they often require solving a system of nonlinear algebraic equations at each integration step, but they allow non-stationary spatial temperature fields to be modeled with sufficient accuracy [4]. In this case, a solution is obtained for rigid systems of differential equations, for which the main criterion is to ensure process stability with lower accuracy. An example of the implementation of both classes of methods under the action of temperature and force loads is the LS-DYNA software package, which allows solving a large group of problems of contact interaction of solid deforming bodies [5–7].

Objective of the work. Using the capabilities of the LS-DYNA software package, develop a version of a substantiated mathematical model of a reinforced concrete protective structure to determine its SSS when concrete with cracks and reinforcement beyond the yield point operate under conditions of emergency force and temperature loads.

Research methodology. An example of the practical application of the capabilities of the specified complex is the solution to the problem of determining the fire resistance limit of the wall of a reinforced concrete storage facility for spent nuclear fuel. A numerical experiment was conducted for the most dangerous cross-section of a precast reinforced concrete structure, the concrete of which was modified with a complex additive Berament A2 [8], which reduces the amount of free water in the pores and increases the protective properties of the material (Fig. 1). Finite element modeling was performed for the fragment and the initial data and parameters of heat exchange processes were specified.



a) *b*)

Fig. 1. Fragment of a reinforced concrete load-bearing wall of a protective structure:
a – the most dangerous cross-section of the wall; *b* – reinforcement diagram of the dangerous cross-section

The selected size of the FE and their number ensure high performance of calculations without reducing their accuracy. When specifying the external load, explicit integration methods were used, and when specifying the fire load, a thermal problem was solved.

Results of the study. The LS-DYNA complex includes several resolvers with an explicit type of integration over time. This method of mathematical modeling allows for the numerical solution of differential equations or other mathematical models based on the calculation of function values at individual points in time or space.

For the Euler method with one variable *(y'=f(t, y))*, the value of the function y at the next step *(t+h)* of integration is determined through the known values of the function at the previous step *(t)* and its derivatives with respect to time and the function at this point according to the expression [9]:

$$y_{n+1} = y_n + h \cdot f(t_n y_n), \tag{1}$$

where: y_{n+1} is the function value at the future time step; y_n is the function value at the current time step; h is the time step; t_n is the time at the current time step; $f(t_n,y_n)$ is the function value f at the point (t_n,y_n), which is known.

The value of y_{n+1}, at the next time step $(t_{n+1} = t_n + h)$, is calculated in four intermediate steps k_1, k_2, k_3 and k_4 and is determined as follows:

calculation of k_1:

$$k_1 = h \cdot f(t_n y_n), \tag{2}$$

calculation of k_2:

$$k_2 = h \cdot f(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}), \tag{3}$$

calculation of k_3:

$$k_3 = h \cdot f(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}), \tag{4}$$

calculation of k_4:

$$k_4 = h \cdot f(t_n + h, y_n + k_3), \tag{5}$$

final calculation:

$$y_n + 1 = y_n + \frac{1}{6} \cdot (k_1 + 2k_2 + 2k_3 + k_4), \tag{6}$$

where: h is the time step, t_n is the time at the current step, y_n s the value of the function at the current time step, $f\,(t, y)$ is the function defining the differential equation, k_1, k_2, k_3, k_4 are intermediate steps that help in calculations.

The calculation of the velocities of the nodes of the FE using the explicit method of integrating dynamic equations is performed using the expression [9–12], which is an explicit numerical method for solving the equations of dynamics:

$$\mathbf{v}^{n+0.5} = \left(\mathbf{u}^{n+1} - \mathbf{u}^{n}\right) / \Delta t^{n+0.5} \Rightarrow \mathbf{u}^{n+1} = \mathbf{u}^{n} + \Delta t^{n+0.5} \mathbf{v}^{n+0.5}. \tag{7}$$

The displacements of the nodes FE are calculated using the expression:

$$\mathbf{x}^{n+1} = \mathbf{x}^{0} + \mathbf{u}^{n+1}. \tag{8}$$

Calculation of the accelerations of the nodes of FE when performing the approximation of time derivatives is the basic expression of the finite difference method:

$$\mathbf{a}^{n} = \left(\mathbf{v}^{n+0.5} - \mathbf{v}^{n-0.5}\right) / \Delta t^{n} \Rightarrow \mathbf{v}^{n+0.5} = \mathbf{v}^{n-0.5} + \Delta t^{n} \mathbf{a}^{n}. \tag{9}$$

When using the expressions written above, equation (1) takes the form:

$$\mathbf{M}\ \mathbf{a}^{n} = \mathbf{F}^{n}; \quad \mathbf{F}^{n} = \sum_{e=1}^{en} \left(\mathbf{F}_e^{ext} - \mathbf{F}_e^{\text{int}}\right). \tag{10}$$

When solving a system of linear algebraic equations, the acceleration of nodes FE is calculated by inverting the matrix M:

$$\mathbf{a}^{n} = \mathbf{M}^{-1} \mathbf{F}^{n}. \tag{11}$$

The time step calculation at this stage is carried out using the Courant-Friedrichs-Lewy number, calculated using the expression:

$$\Delta t \le \Delta t_{crit} = \min \frac{l_e}{c_e}, \tag{12}$$

where: c_e is the value obtained from the dependence: $c_e = \sqrt{E_e/\rho_e}$; l_e is the spatial step of the applied FE mesh.

Mathematical models of dynamics and SSS of the structural system are implemented in the calculation scheme, which takes into account the displacements of a rigid deformable body (RDB), simultaneously with the motion of the deformable one from the time t = 0 to a given time t. In the diagram, the RDB in the initial state is marked as Ω0. In the initial position, the boundary surface of the body is designated as Γ0. At a given time t , the current position and geometric configuration of the RDB is designated by Ω with the boundary surface denoted by Γ. During the period of motion of a body with the initial configuration Ω0 from the initial position to the position and configuration Ω, existing at the current time, a certain point, having the corresponding set of coordinates X in the initial state and belonging to the region Ω0, will move and be localized in the region Ω0 with a new set of coordinates x.

In order to describe the dynamics of the RDB interaction, the basic equation of motion (Fig. 2) taking into account a set of conservative laws of dynamics is adopted in accordance with the works of Belichko, Bailey, etc. [1-4]. Taking into account the accepted notations, the equation of the impulse balance, adopted as the basic one, has the form:

$$\sigma_{ij,i} + \rho \cdot f_i = \rho \cdot \ddot{x}_i\,, \tag{13}$$

where: $\sigma_{ij,i}$ is the Cauchy stress tensor of RDB at a given point; ρ – is the density of the material of RDB at a given point; $\rho \cdot f_i$ are the external forces applied to the body at a given point of RDB; $\ddot{x}_i$ is the acceleration of a given point RDB.

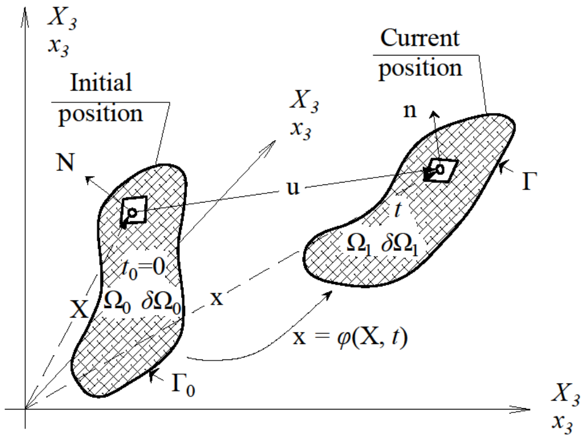


Fig.2. Initial and current position of a solid body deformed during motion

The equation of conservation of RDB mass is written as a dependence:

$$\rho \cdot \det(\mathbf{J}) = \rho_0\,, \tag{14}$$

where: ρ_0 is the density of the material in the initial state at a given RDB point; $\det(\mathbf{J})$ is the determinant of the predicted tangent matrix.

The equation for the energy balance law is written in terms of kinetic energy and potential internal energy, the sum of which is equal to the sum of the work of external forces.

$$P^{int} + P^{kin} = P^{ext} + P^{heat}\,. \tag{15}$$

Kinetic energy can be determined by the expression:

$$P^{kin} = 0.5\frac{d}{dt}\int_{\Omega} \rho \mathbf{v} \cdot \mathbf{v} d\Omega\,. \tag{16}$$

Internal energy is calculated by the relationship:

$$P^{ext} = \int_{\Omega} \mathbf{v} \cdot \rho \mathbf{b} d\Omega + \int_{\Gamma} \mathbf{v} \cdot \mathbf{t} d\Gamma \quad (17)$$

According to the works [1–4], in the absence of a source of thermal energy, the form of the energy conservation equation is as follows:

$$\frac{d}{dt}\int_{\Omega} \rho w^{\text{int}} + (0.5\rho \mathbf{v} \cdot \mathbf{v}) d\Omega = \int_{\Omega} \mathbf{v} \cdot \rho \mathbf{b} d\Omega + \int_{\Gamma} \mathbf{v} \cdot \mathbf{t} d\Gamma . \quad (18)$$

The energy balance equation taking into account the deformation of RDB takes the form:

$$\rho \dot{w}^{int} = 0.5\sigma_{ij}\left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right]. \quad (19)$$

The limiting conditions for the limitation of the motion of RDB Γ_f are written in the form:

$$\sigma_{ij} n_j = t_i(t), \quad (20)$$

where n_j is the normal to the limiting surface of RDB, which is necessarily directed outward.

To set the limit conditions for the deformation parameters on the boundary surface of RDB, we write the expression:

$$x_i(\mathbf{X},t) = \bar{x}_i(t) . \quad (21)$$

In contact interaction between RDB, we write down the given boundary conditions:

$$\left(\sigma_{ij}^{+} - \sigma_{ij}^{-}\right) n_j = 0 . \quad (22)$$

For bodies interacting with each other, their current state on virtual displacements δx_i can be written in the form of a work balance equation:

$$\int_{\Omega} \left[\rho\, \ddot{x}_i + \sigma_{ij,j} - \rho\, f_i\right] \delta x_i d\Omega + \int_{\Gamma_f} \left[\sigma_{ij} n_j - t_i\right] \delta x_i d\Gamma + \int_{\Gamma_c} \left(\sigma_{ij}^{+} - \sigma_{ij}^{-}\right) n_j \delta x_i d\Gamma = 0. \quad (23)$$

Expression (18) takes on its final form [1–4] after writing the sum of the virtual values of work, which is equivalent to zero, and carrying out the appropriate transformations:

$$\int_{\Omega} \rho\, \ddot{x}_i \delta x_i d\Omega + \int_{\Omega} \sigma_{ij,j} \delta x_i d\Omega - \int_{\Omega} \rho\, f_i \delta x_i d\Omega - \int_{\Gamma_f} t_i \delta x_i d\Gamma - \int_{\Gamma_c} t_i^c \delta x_i d\Gamma = 0 . \quad (24)$$

The finite element approximation of the main equations of dynamic interaction is implemented using a dependence that describes the interpolation process within the internal limits of the FE space of parameter distributions in the following form:

$$x_i(\mathbf{X},t) = \bar{x}_i(\mathbf{X}(\xi,\eta,\zeta),t) = \sum_{j=1}^{m} \phi_j(\xi,\eta,\zeta) x_i^j(t) . \quad (25)$$

where m is the number of nodes corresponding to the type and shape of FE; ϕ_j is the shape function in parametric form (parameters ξ, η, ζ); x_i^j is the current coordinate of the FE node on the corresponding axis.

For virtual displacements of a mechanical system for FE, the potential energy is determined from the equation:

$$\delta \Pi_e = \int_{\Omega_e} \rho\, \ddot{x}_i \Phi_i^e d\Omega + \int_{\Omega_e} \sigma_{ij} \Phi_{ij}^e d\Omega - \int_{\Omega_e} \rho\, f_i \Phi_i^e d\Omega - \int_{\Gamma_e} t_i \Phi_i^e d\Gamma , \quad (26)$$

where $\Phi_i^e = (\phi_1, \phi_2, ..., \phi_k)_i^e$.

For the complete set of FE [2], the principle of possible displacements is taken into account. In this case, the general equation of conservation of energy of the RDB, discretized onto the FE mesh, is written by the expression:

$$\sum_{e=1}^{en}\left[\int_{\Omega_e}\rho\ \ddot{x}_i\Phi_i^e d\Omega+\int_{\Omega_e}\sigma_{ij}\Phi_{ij}^e d\Omega-\int_{\Omega_e}\rho\ \ f_i\Phi_i^e d\Omega-\int_{\Gamma_e}t_i\Phi_i^e d\Gamma\right]=0 \quad (27)$$

In matrix form, expression (24) takes the following form:

$$\sum_{e=1}^{en}\left[\int_{\Omega_e}\rho\,\mathbf{N}^{\mathrm{T}}\mathbf{N}\ \mathbf{a}_e d\Omega+\int_{\Omega_e}\mathbf{B}^{\mathrm{T}}\boldsymbol{\sigma}\ d\Omega-\int_{\Omega_e}\rho\,\mathbf{N}^{\mathrm{T}}\mathbf{b}\ d\Omega-\int_{\Gamma_e}\mathbf{N}^{\mathrm{T}}\mathbf{t}\ d\Gamma\right]=0, \quad (28)$$

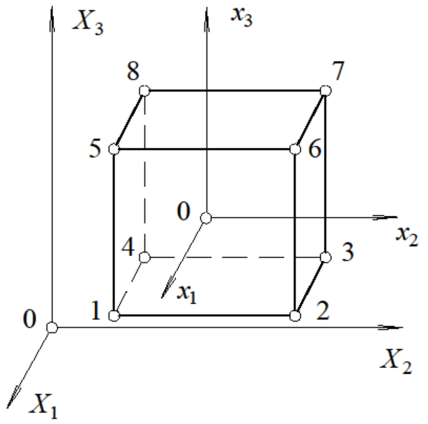
where: N is the matrix of parametric interpolation functions corresponding to the shape and type of the FE; is the stiffness matrix; σ is the stress vector; a_e – is the acceleration vector of the FE nodes; b is the loading vector; t is the traction force vector..

Eight-node FE massive hexahedral type SOLID (Fig. 3) are adopted for modeling the behavior of reinforced concrete structures from the PC base. The equation for determining the coordinates of the nodes of form (26) for a given FE has the form:

$$x_i(\mathbf{X},t)=\bar{x}_i(\mathbf{X}(\xi,\eta,\zeta),t)=\sum_{j=1}^{8}\phi_j(\xi,\eta,\zeta)x_i^j(t), \quad (29)$$

where ϕ_j is the parametric function of the form for the *j*-th node of the FE of a given type:

$$\phi_j=0.125(1+\xi\xi_j)(1+\eta\eta_j)(1+\zeta\zeta_j), \quad (30)$$



Node	ξ	η	ζ
1	-1	-1	-1
2	1	-1	-1
3	1	1	-1
4	-1	1	-1
5	-1	-1	1
6	1	-1	1
7	1	1	1
8	-1	1	1

Fig. 3. Geometry of a hexahedral FE of the SOLID type with eight nodes

Parameters ξ_j, η_j, ζ_j are adopted in accordance with the calculation scheme.

The interpolation matrix for this type of FE has the following form:

$$\mathbf{N}(\xi,\eta,\zeta)=\begin{bmatrix}\phi_1 & 0 & 0 & \phi_2 & 0 & \cdots & 0 & 0\\ 0 & \phi_1 & 0 & 0 & \phi_2 & \cdots & \phi_8 & 0\\ 0 & 0 & \phi_1 & 0 & 0 & \cdots & 0 & \phi_8\end{bmatrix}. \quad (31)$$

The expression for the stress vector is written as:

$$\boldsymbol{\sigma}=\begin{pmatrix}\sigma_x & \sigma_y & \sigma_z & \tau_{xy} & \tau_{yz} & \tau_{zx}\end{pmatrix}^{\mathrm{T}}. \quad (32)$$

This approach allows taking into account the plastic deformation of the power component of thermal and power loads. To obtain a complete picture of the SSS of a reinforced concrete protective structure, a heat engineering problem is solved.

Method for solving the thermal problem. To solve the problem of temperature distribution of parts of structural systems under the influence of standard fire temperature conditions, a non-stationary differential equation of thermal conductivity is used in the form:

$$c_p(\theta)\rho(\theta)\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left(\lambda(\theta)\frac{\partial\theta}{\partial y}\right)+\frac{\partial}{\partial x}\left(\lambda(\theta)\frac{\partial\theta}{\partial y}\right), \quad (33)$$

where: $c_p(\theta)$ is the specific heat capacity of concrete, depending on the temperature θ; $\lambda(\theta)$ is the thermal conductivity coefficient of concrete, depending on the temperature θ; $\rho(\theta)$ is the density, depending on the temperature θ.

Approximation of the differential heat conduction equation using the FE method is performed using the dependence [5, 13–16]:

$$[\mathbf{C}_e]\{\theta_e\} + [\mathbf{K}_e]\{\theta_e\} = \{Q_e\}. \tag{34}$$

he components of the approximation equation are determined by the following dependencies:

Heat capacity matrix of the FE: $[\mathbf{C}_e] = \rho \cdot C_p \int_V \{\mathbf{N}\} dV$.

Thermal conductivity matrix of the FE: $[\mathbf{K}_e] = \int_V [\mathbf{B}]^T [\mathbf{D}] [\mathbf{B}]\, dV$.

Temperature interpolation matrix in the FE volume: $[\mathbf{B}] = \{L\}\{N\}\theta$.
Average temperature in the volume of the FE: $\theta = \{N\}^T \{\theta_e\}$.
Vector of temperature indicators in the nodes of the FE: $\{\theta_e\}$.

Matrix of thermal conductivity coefficients: $[\mathbf{D}] = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$.

Vector differential operator: $\{L\} = \left\{ \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \right\}^T$.

The matrix of heat flows on the faces of the FE: $\{Q_e\} = q_W \int_S \{N\} ds$.

Vector operator of interpolation of parameters in the volume of the FE: $\{N\}$.

To approximate the non-stationary differential equation of heat conductivity (34) for determining temperature indicators, a system of non-linear algebraic equations is used, which are written in matrix form as the expression:

$$[\mathbf{K}]\{\theta_e\} = \{Q_e\}, \tag{35}$$

де: $[\mathbf{K}]$ is the matrix of equivalent thermal conductivity, depending on all thermophysical characteristics: specific heat capacity, thermal conductivity coefficient and density of materials of parts of structural systems.

Then the system of algebraic nonlinear equations (35) in the most general form:

$$\{P(\theta)\} = \{Q_e\}, \tag{36}$$

where: $\{P(\theta)\}$is a vector that determines the conditions of internal heat flows in the nodes of the FE and is written according to the values of the heat flux density on the faces of the FE.

The root systems of algebraic nonlinear equations (35) are calculated by iterative procedures of the Newton-Raphson method [2, 3, 5, 10, 13-16]. The purpose of performing these operations is to minimize errors and violations. Symbolically, this algorithm can be represented as follows:

$$\{\Phi\} \equiv \{Q_e\} - \{P(\theta)\} \rightarrow \{0\}. \tag{37}$$

The implementation of the Newton-Raphson method becomes possible when using a truncated Taylor series with a permanent calculation of the error vector remaining in the remainder. This method of implementation allows solving linear algebraic systems of equations to obtain a numerical value at a given iteration step. Under these conditions, the system of equations (37) is written as a matrix equation:

$$\left[\mathbf{K}_T^{(i-1)}\right]\left\{\Delta\theta_e^{(i)}\right\} = \left\{Q^{(i)}\right\} - \left\{P^{(i)}\right\}. \tag{38}$$

In compliance with the method described above, we perform equilibrium iterations (i = 1, 2, 3, ...). The result of performing these mathematical operations are new temperature values at the current step, which is achieved by calculating in accordance with the dependence:

$$\{\theta^{(i)}\} = \{\theta^{(i-1)}\} + \{\Delta\theta^{(i)}\}. \tag{39}$$

The process of performing iterations continues until an acceptable convergence is achieved, meeting the requirements of the established accuracy of the results. Equation (38) contains the coefficients of the tangent matrix [$\mathbf{K}_T$]. The data are calculated using the expression:

$$\left[\mathbf{K}_T^{(i-1)}\right] \equiv \left(\frac{d\{\Phi\}}{d\{\theta\}}\right)_{i-1}. \tag{40}$$

To write the vector {Φ} as a truncated Taylor series, the expression below is applicable:

$$\{\Phi^{(i)}\} \cong \{\Phi^{(i-1)}\} + \left[\mathbf{K}_T^{(i-1)}\right]\{\Delta\theta^{(i)}\}, \tag{41}$$

where: $\{\Delta\theta^{(i)}\} = \{\theta^{(i)}\} + \{\theta^{(i-1)}\}$ is the temperature increment vector for further iteration.

Equation (40) contains {$Q^{(i)}$}, which is a vector of heat flux values in the FE nodes, which are calculated by overwriting the new values of the external heat flux vector. The latter is set by adding the corresponding temperature increment determined by the specified fire temperature regime and the specified heat exchange coefficients {Q_{0n}}. When using these data, the following components are updated: the equivalent thermal conductivity matrix [**K**], which depends on the temperatures determined at the previous integration step; the equivalent vector of internal heat flux values at the FE nodes {P(θ)},obtained by updating the equivalent thermal conductivity matrix. The parameters are updated by numerical integration of the vector over time using the Euler method according to the dependence:

$$\{\theta_{n+1}\} - \{\theta_n\} = \Delta t_n (1-\varsigma)\{\dot{\theta}_n\} + \Delta t_n \varsigma \{\dot{\theta}_{n+1}\}, \tag{42}$$

where: Δt_n is the integration step over time;

ζ is the Euler exponent for the selected integration scheme (the Crank-Nicholson scheme), equal to 0.5.

The matrix of equivalent thermal conductivity [**K**] is expressed, taking into account the application of this scheme, by the dependence:

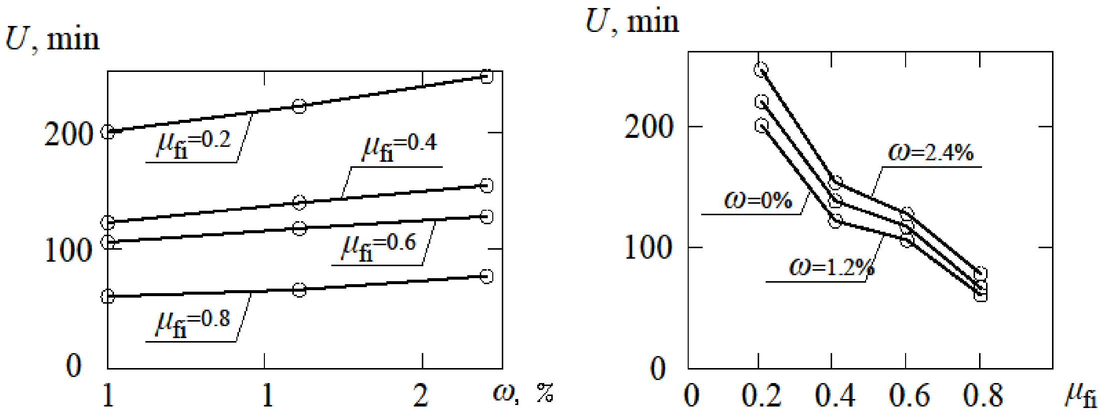
$$[\mathbf{K}] = \frac{1}{\varsigma \Delta t_n}[\mathbf{C}_e] + [\mathbf{K}_e], \tag{43}$$

To update the vector of boundary heat fluxes {Q(θ)}according to the Crank-Nicholson scheme, the following dependence is used:

$$\{Q_n(\theta)\} = \{Q_e\} + \frac{1-\varsigma}{\varsigma}[\mathbf{C}_e]\{[\dot{\theta}]_n\} - [\mathbf{K}_e]\{\theta_n\}. \tag{44}$$

When setting the boundary value problem of heat conductivity, the third-order limit conditions and the standard fire temperature regime were used [17, 18]. Based on the results of the numerical experiment on heating a reinforced concrete wall for 180 minutes under the standard fire temperature conditions, the results of temperature distribution in the wall of the protective structure were obtained. Thus, the fire resistance limits of the reinforced concrete wall of the storage facility accepted for the research were determined.

Regularities of changes in the fire resistance limit of a reinforced concrete wall depending on the loading and mass fraction of the Berament A2 additive were established. The graphs identified patterns are given at Fig.4. The graphs of the identified patterns are presented in Fig. 4. They show that the patterns expressing the dependence of the fire resistance limit on the mass fraction of the Berament A2 additive and the load level are linear.



a) b)

Fig. 4. Graphs of the dependence of the fire resistance limit of a reinforced concrete wall on:
a – load factor; b – mass fraction of the additive to concrete Berament A2

In this case, the dependence of the fire resistance limit of a reinforced concrete wall on the mass fraction of the Berament A2 additive is proportional, and the dependence of the fire resistance limit of a reinforced concrete slab on the load level is inversely proportional.

Scientific novelty and practical value. Developed on the basis of the capabilities of the LS-DYNA software package and substantiated mathematical models and algorithms for taking into account the features of the action of emergency thermal power loads on reinforced concrete protective structures, models of the operation of reinforced concrete components with plastic deformations in the reinforcement and cracks in the concrete under the action of such loads allow us to determine the stress-strain state of the structure until the plastic state is reached.

As a result of the experiment, it was established that the fire resistance limit for the boundary state of the thermal insulation capacity does not occur during the established time, and the fire resistance limit for the bearing capacity at a load level of 0.4Qmax for a reinforced concrete wall was 276 min. The regularities of the dependence of the fire resistance limit of a reinforced concrete wall on the load level and the mass fraction of the Berament A2 additive were revealed, which are linear in nature, and its fire resistance increases with an increase in the mass fraction of the additive from 0 to 2.4%.

Conclusions. The above-stated substantiated methodology using explicit and elements of implicit methods allowed us to correctly solve the problem, taking into account the nonlinear deformation of reinforced concrete structure materials and spatial temperature fields from external temperature load. This approach allows simulating emergency dynamic temperature and force loads on protective structures, such as a drone strike followed by an explosion and others.

References

[1] I.A.Yakovenko, "Transformatsiynyy element, shcho pov″yazuye zalezhnosti mekhaniky ruynuvannya z teoriyeyu zalizobetonu", *Nauka ta budivnytstvo*, no. 4(18), pp. 28–37, 2018. https://doi.org/10.33644/scienceandconstruction. v18i4.54.

[2] LS-DYNA Theory Manual. Livermore Software Technology (Lst), An Ansys Company, California, Michigan, 2022.

[3] T. Belytschko, R.L. Chiapetta & H.D. Bartel, "Efficient Large Scale Non-Linear Transient Analysis by Finite Elements", *International Journal for Numerical Methods in Engineering*, 10 (1976), 1, pp. 579–596.

[4] C.G. Bailey, "Membrane Action of Slab/Beam Composite Floor Systems in Fire", *Engrg. Struct.*, 26(12), pp. 1691–1703, 2004.

[5] L. Reese et. al., "Field Tests and Numerical Modeling of Vehicle Impacts on a Boulder Embedded in Compacted Fil", *International Journal of Protective Structures*, vol. 5, no. 4, pp. 435–451, 2014.

[6] Long T. Phan, Therese P. McAllister, John L. Gross, Morgan J. Hurley, *Best Practice Guidelines for Structural Fire Resistance Design of Concrete and Steel Buildings*. NIST Technical Note, 2010.

[7] D. Marzougui, C.D. Kan, R. Samaha, C. Cui, L. Nix, "Extended Validation of the Finitel for the 2007 Chevrolet Silverado Pick-Up Truck (MASH 2270kg Vehicle)", *Presented at the Transportation Research Board 92nd Annual Meeting,* Washington, D.C., 2013.

[8] I. Karkhut and R. Havryshkiv, "Study of the Strength of Concretes Based on M500 Cements with the Addition of Berament A2 at an Early Age", *Lecture Notes in Civil Engineering. Proceedings of CEE 2023*. Springer, 2024. DOI: 10.10007/978-3-031-44935-0-17.

[9] J.O**.** Hallquist, *LS-DYNA Theory Manual*. *Livermore Software Technology Corporation*: California, USA. 2005.

[10] L. Lim, A.H. Buchanan, P.J. Moss and J-M. Franssen, "Computer Modeling of Restrained Reinforced Concrete Slabs in Fire Conditions", *ASCE Journal of Structural Engineering*, 130:12, pp. 1964–1971, 2004.

[11] A.S. Sakharov, V.M. Kyslookyi, V.V. Kyrychevskyi, I.A. Altenbakha, *Metod skinchennykh эlementiv u mekhanitsi tverdykh til*. Kyiv: Vyshcha shkola, 1982.

[12] Methode de prevision par le calcul du comportement au feu des structures en beton : Cakier du CSTB, no. 1270, pp. 34–56, 2001.

[13] A computer program for analysis of structures at elevated temperature conditions. Liege, Belgium. University of Liege, 2022.

[14] ANSYS, ANSYS 9.0 Manual Set, ANSYS Inc., Southpoint, 275 Technology Drive, Canonsburg, PA 15317, USA.

[15] Competitive Steel Buildings Through Natural Fire Safety Concept, Draft Final Report. Part 2. Franssen J-M., et al. Luxembourg : Profil Arbed Centre de Recherches, 1999.

[16] Milke J.A. *Analytical Methods for Determining Fire Resistance of Steel Members*: SFPE Handbook of Fire Protection Engineering. 3rd ed. Di Nenno, ed., Quincy, Mass.: National Fire Protection Association, 2002.

[17] S.V. Pozdeev, "Rozvytok naukovykh osnov vyznachennya mezh vohnestiykosti nesuchykh zalizobetonnykh konstruktsiy", dys. ... dokt. tekhn. Nauk: Cherkasy, 2012.

[18] T.M. Shnalʹ, "Rozvytok naukovykh osnov rozrakhunkovoyi otsinky vohnestiykosti budivelʹnykh konstruktsiy za umov vplyvu parametrychnykh temperaturnykh rezhymiv pozhezh", dys. ... dokt. tekhn. Nauk: Lʹviv, 2019.

ОСОБЛИВОСТІ МОДЕЛЮВАННЯ ЗАЛІЗОБЕТОННИХ ЗАХИСНИХ СПОРУД ЯВНИМ МЕТОДОМ ПРИ РОЗРАХУНКАХ НА ТЕМПЕРАТУРНОСИЛОВІ НАВАНТАЖЕННЯ

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Анотація. Математичне моделювання сьогодні є основою наближених методів розрахунків та визначення напружено-деформованого стану (НДС) конструкцій при температурних впливах. Воно дозволяє чисельними методами скінченних елементів (СЕ) отримати обґрунтовані розв'язки багатьох складних задач у випадках дії силових та температурних навантажень на статично невизначені залізобетонні конструкції, в тому числі з врахуванням пластичних деформацій та нестаціонарних тривимірних температурних полів. В статті описані основні етапи моделювання явним методом залізобетонних захисних споруд при силових навантаженнях та особливості теплової задачі, які базуються на можливостях програмного комплексу LS-DYNA. Описано алгоритми математичного моделювання з детальним покроковим обґрунтуванням застосованих залежностей явного методу. Вказано, що правильний вибір критеріїв взаємодії та обґрунтованих моделей на підставі аналізу конструкції дозволяє отримати адекватні результати чисельного експерименту, які підтверджені іншими дослідниками. Наведені залежності, які дозволяють обчислювати значення функції на майбутньому кроці часу з використанням вже відомих значень функції на поточному кроці та її похідних. Розрахунок швидкостей вузлів СЕ при використанні явного методу інтегрування динамічних рівнянь виконується із застосуванням виразу, який є явним числовим методом розв'язку рівнянь динаміки. Наведено базовий вираз для розрахунку прискорень вузлів СЕ за умов виконання апроксимації похідних за часом методом кінцевих різниць.

Для повного набору СЕ враховано принципові можливі переміщення вузлів, узагальнене рівняння збереження енергії твердого деформівного тіла, яке дискретно накладене на сітку СЕ.

Для випадку дії температурних навантажень в режимі пожежі наведено підхід до вирішення теплової задачі. Показано, що викладена обґрунтована методика з використанням елементів явного та неявного методів дозволяє коректно розв'язати поставлену теплову задачу з врахуванням нелінійного деформування матеріалів залізобетонної захисної конструкції та просторових температурних полів від зовнішнього температурного навантаження.

Ключові слова: явний метод інтегрування, пожежні навантаження, гексаедральний скінченний елемент, залізобетонні конструкції, теплова задача.

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