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THREE-FACTOR EXPERIMENTAL MATRIX AND LOW-CYCLE TESTING METHODOLOGY FOR CONCRETE BEAMS WITH WORKING GFRP REINFORCEMENT IN THE TENSION ZONE

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Abstract. Published repeated-loading programmes for GFRP-reinforced flexural members remain methodologically heterogeneous with respect to cycle limits, number of repetitions, load normalisation, and monitored response variables, which limits direct comparison between datasets. The aim of this methodology paper is to develop and substantiate a formalised experimental programme for concrete beams with working longitudinal GFRP reinforcement in the tension zone under prescribed low-cycle loading regimes. The scientific result is the experimental procedure itself rather than quantitative results from the beam series. The programme comprises 15 specimens and three factors: working GFRP reinforcement level X_1, concrete strength class X_2, and prescribed low-cycle loading regime X_3. The matrix follows the geometric layout of a face-centred central composite design and comprises eight factorial corner points, six face-centred axial points, and one centre point. Reinforcement arrangements 3Ø8, 3Ø12, and 2Ø18 correspond to actual $\mu_f=A_f/(bh)$ values of 0.503, 1.130, and 1.697%, while X_2 comprises C16/20, C20/25, and C25/30. X_3 is treated as an ordered composite regime because its levels simultaneously change M_{min}, M_{max}, mean moment, amplitude, and the load ratio R. Cyclic demand is normalised with respect to the a priori predicted laboratory ultimate flexural capacity $M_{u,pred}$, evaluated from strain compatibility and equilibrium of the normal section. Three regimes are prescribed: $0\leftrightarrow0.4M_{u,pred}$, $0.4\leftrightarrow0.6M_{u,pred}$, and $0.6\leftrightarrow0.8M_{u,pred}$, each comprising ten cycles with 10-min holding periods at the upper and lower limits. Four-point bending is used with a 980 mm support span, a 415 mm shear span and a 150 mm constant-moment region; the bending moment is related to the total jack force by M=0.2075P. Instrumentation combines midspan deflection measurement, continuous concrete and GFRP strain acquisition at 62.5 Hz, and manual measurement of the maximum opening of a flexural crack. Predefined procedures are provided for averaging paired GFRP strain gauges, fitting the strain profile ε(y), calculating curvature and neutral-axis position, and evaluating cyclic secant stiffness. Because no identical factor combinations are replicated and only one centre point is available, the matrix is intended for direct comparisons, trend analysis, and exploratory modelling rather than a formal lack-of-fit assessment or a fully validated quadratic response surface. The methodological novelty lies in integrating the factor matrix, composite cyclic regimes, through-depth strain monitoring, and predefined interpretation rules within one protocol.

Keywords: GFRP reinforcement, concrete beam, low-cycle loading, experimental matrix, deflection, strain, crack opening, cyclic stiffness.

Introduction. Glass fibre-reinforced polymer (GFRP) reinforcement is used in concrete structures due to its high tensile strength, low density and corrosion resistance. At the same time, its modulus of elasticity is significantly lower than that of reinforcing steel, and its behaviour prior to failure is predominantly linearly elastic. Therefore, following the formation of flexural cracks, deflections, crack opening widths, material strains and changes in bending stiffness become particularly important for GFRP-reinforced flexural members [1–6].

Under repeated loading, the mechanical state of a beam depends not only on the maximum level of the cycle, but also on its lower limit, average level, amplitude, the load ratio R and the previous loading–unloading history. Differences between published cyclic programmes complicate a direct comparison of results [7–10]. It is therefore important to develop a formalised procedure that simultaneously defines the factors, the cyclic protocol, the calculation basis for standardisation, the measurement system and the rules for subsequent data processing.

Analysis of recent research and publications. Thériault and Benmokrane [1] demonstrated the combined significance of the FRP reinforcement ratio and concrete strength for flexural behaviour. Toutanji and Saafi [2], Mota, Alminar and Svecova [3], El-Nemr, Ahmed and Benmokrane [4], Saleh et al. [5], and Gouda, Hassanein and Galal [6] confirmed that, for GFRP-reinforced beams, the load-bearing capacity must be considered in conjunction with serviceability criteria. This justifies the selection of the level of longitudinal reinforcement and the concrete class as separate factors.

The low-cycle and fatigue behaviour of GFRP-reinforced beams was investigated by Wu et al. [7, 8], Nagy, Asadian and Galal [9], and Murthy et al. [10]. In these studies, deflections, cracks, deformations, stiffness and/or bar slippage were monitored; however, different numbers of cycles, levels of P_{min} and P_{max}, load ratios R and control methods were employed. Thus, the existing body of work is sufficient to justify the measured parameters, but does not constitute a single standardised low-cycle testing protocol.

The closest Ukrainian counterpart is the work by Koval and Hrymak [11, 12] on BFRP-reinforced beams involving ten cycles followed by loading to failure. Their results cannot be quantitatively extrapolated to GFRP, but they confirm the validity of a separate analysis of deformability and crack parameters. In the work by Klymenko, Grynova and Gerak [13], a different 15-specimen test programme is proposed for beams with combined GFRP/steel longitudinal reinforcement, different concrete grades and different loading principles. The present programme constitutes a separate, modified experimental series and does not represent a repetition of the data or the factorial structure [13]. The main differences between the two methodological approaches are presented in Table 1.

Table 1 – Differences from the methodological programme by Klymenko, Grynova and Gerak [13]

Feature	**Work [13]**	**Current methodology**
Longitudinal reinforcement	Combined GFRP/steel	GFRP working reinforcement in the tension zone; top 2Ø6 A240 – steel
Reinforcement factor	Proportion of GFRP in combined reinforcement	3Ø8, 3Ø12, 2Ø18; actual A_f, μ_f, ρ_f
Concrete	C12/15; C16/20; C25/30	C16/20; C20/25; C25/30
Load standardisation	Relative to M_{cr}	Relative to the predicted $M_{u,pred}$
Cyclic protocol	Other loading principles; holding times 5–10 mins	Three ordered low-cycle loading regimes; 10 cycles; 10-minute dwell times at the limits
Crack inspection	Marking on two faces, MPB-2	One face; maximum flexural crack; 0.1 mm template; no mapping
Deformation monitoring	Alternative measurement scheme	7 concrete channels + 2 sensors on each GFRP rod; 62.5 Hz
Statistical formulation	Preliminary three-factor formulation	Matrix based solely on FCCD geometry; one central point; no estimation of pure error or lack-of-fit test

Provision [13] regarding control samples ‘from the same batch’ relates to that previous programme. In the current series, the beams are concreted within a single production day; more than one batch is used for each concrete class, but the exact link between a specific beam and a specific batch is not recorded in the original documentation. Therefore, such a link is not reconstructed retrospectively and is not used as a hidden statistical block.

Experimental designs of the central composite design type allow for an efficient coverage of the factor space; however, their statistical interpretation depends on the nature of the factors and the presence of replicates [14–16]. In this work, the design is used primarily as a geometric arrangement of 15 destructive tests.

Purpose and objectives. The aim of this work is to develop and provide a methodological justification for a programme of experimental research into the stress-strain state of concrete beams with working longitudinal GFRP reinforcement in the tension zone under ordered low-cycle loading regimes, followed by the determination of the post-cyclic ultimate flexural capacity.

To achieve this aim, it is necessary to: justify factors X_1–X_3; formulate an experimental matrix; determine the a priori Mu,pred; formalise the four-point test setup and three cyclic test protocols; establish a measurement system and data processing algorithms; distinguish between the cyclic and post-cycle stages; and define the statistical and instrumental limits of interpretation.

Materials and methods. The test beams have a cross-section of b = 120 mm, h = 250 mm and a length of l = 1180 mm. The axis of the working GFRP reinforcement in the tension zone is situated 25 mm from the bottom face, hence h_0= d = 225 mm. The distance from the side surface of the concrete to the surface of the outermost GFRP bar is 21 mm. The upper longitudinal steel reinforcement consists of 2 × Ø6 A240 bars, whilst the transverse reinforcement consists of Ø6 A240 bars spaced at 50 mm intervals. The longitudinal working GFRP bars manufactured by IMPERATYV are straight, 1180 mm long, with spiral winding and a sand coating. The specified E_f= 50.000 MPa; f_{fu} for Ø8, Ø12 and Ø18 is 1033.8; 831.9; 885.8 MPa respectively. The passport number for the specific batch is not recorded in the available documentation; in-house tensile tests on the batch to determine these characteristics have not been carried out. The material requirements for composite reinforcement are specified in DSTU 9065:2021 [17].

$$\mu f = A_f/(\mathrm{bh}) \cdot 100\ \%, \quad (1)$$

where μ_f – GFRP reinforcement ratio;
A_f – total cross-sectional area of the working GFRP reinforcement;
b – the width of the beam cross-section;
h – total height of the cross-section.

$$\rho f = A_f/(bh_0) \cdot 100\ \%, \quad (2)$$

where ρ_f – reinforcement ratio relative to the effective depth of the cross-section;
A_f – area of the effective GFRP reinforcement;
b – width of the cross-section;
h_0 – effective height of the cross-section.

The actual and coded levels of the three factors are summarised in Table 2.

Table 2 – Factors and levels of the experimental programme

Factor	−1	0	+1
X_1	3Ø8; A_f =151 mm²; μ_f =0.503 %; ρ_f =0.559 %	3Ø12; A_f =339 mm²; μ_f =1.130 %; ρ_f =1.256 %	2Ø18; A_f = 509 mm²; μ_f= 1.697 %; ρ_f= 1.885 %
X_2	C16/20	C20/25	C25/30
X_3	0↔0.4$M_{u,pred}$	0.4↔0.6$M_{u,pred}$	0.6↔0.8$M_{u,pred}$

The X_3 factor is defined as the ordered low-cycle loading regime. It is described using the mean moment, amplitude and load ratio R:

$$M_m = (M_{max} + M_{min})/2, \quad (3)$$

where M_m is the mean bending moment of the cycle;
M_{max} and M_{min} are the upper and lower limits of the bending moment, respectively.

$$M_a = (M_{max} - M_{min})/2, \quad (4)$$

where M_a is the amplitude of the bending moment;
M_{max} and M_{min} are the upper and lower limits of the bending moment.

$$R = M_{min}/M_{max}, \quad (5)$$

where R is the load-cycle asymmetry coefficient;
M_{min} is the lower limit, and M_{max} is the upper limit of the bending moment.

For $X_3 = -1$: $M_m = 0.20M_{u,pred}$, $M_a = 0.20M_{u,pred}$, R = 0; for $X_3 = 0$: $M_m = 0.50M_{u,pred}$, $M_a = 0.10M_{u,pred}$, R = 0.667; for $X_3 = +1$: $M_m = 0.70M_{u,pred}$, $M_a = 0.10M_{u,pred}$, R = 0.75. Therefore, X_3 is not a standard continuous 'load level' variable. The complete 15-point experimental matrix is given in Table 3.

Table 3 – Experimental matrix

Beam	Code	X_1	X_2	X_3
B1	B111	+1	+1	+1
B2	B11−1	+1	+1	−1
B3	B1−11	+1	−1	+1
B4	B−111	−1	+1	+1
B5	B1−1−1	+1	−1	−1
B6	B−11−1	−1	+1	−1
B7	B−1−11	−1	−1	+1
B8	B−1−1−1	−1	−1	−1
B9	B100	+1	0	0
B10	B−100	−1	0	0
B11	B010	0	+1	0
B12	B0−10	0	−1	0
B13	B001	0	0	+1
B14	B00−1	0	0	−1
B15	B000	0	0	0

The matrix contains 8 corner, 6 axial and 1 centre point and is formed according to the geometry of the face-centred central composite design (FCCD). The very phrase 'based on the FCCD geometry' is fundamental: the programme does not contain replicated central points, and X_3 has a composite categorical nature; therefore, the statistical properties of the classical continuous FCCD are not automatically transferred [14–16].

For each concrete class, three prisms measuring 150×150×600 mm are provided for compression testing, three prisms measuring 150×150×600 mm for determining flexural tensile strength, and three cubes measuring 150×150×150 mm for compression testing, making a total of 27 test specimens [18, 19]. The beams are the beams are cast within a single production day and cured in a closed room under normal temperature and humidity conditions for 28 days. As the exact correspondence between an individual beam and a specific concrete batch is not documented, X_2 is retained as a concrete class factor in the initial statistical analysis. The results of the control tests characterise the concrete at the level of the corresponding class; where necessary, the average prism value of three control prisms of this class is used for post-experimental verification of the calculation, but no 'individual' strength is attributed to a single beam without a documented 'beam–batch' relationship.

The a priori value $M_{u,pred}$ is used only for the normalisation of cyclic loading. It is not the cracking moment M_{cr} and is not equated with the design ultimate flexural capacity M_{Rd}. The calculation is carried out based on the compatibility of deformations and the equilibrium of the normal cross-section in accordance with the principles of DSTU-N B V.2.6-185:2012 [20], DBN V.2.6-98:2009 [21] and current ACI provisions regarding GFRP-reinforced concrete [22]. In the approved a priori calculation, $h_0 = 225$ mm is adopted; the area of the top steel reinforcement $A'_s = 56.55$ mm²,

its distance from the top face d′ = 25 mm, E_s= 210.000 MPa and the ultimate stress 240 MPa. For concrete, f_c= 15.0; 18.5; 22.0 MPa are used, whilst the ultimate compressive strains adopted in this preliminary calculation are ε_{cu}= 3.73; 3.46; 3.20‰ for C16/20, C20/25 and C25/30 respectively. This set of parameters relates to a pre-test prediction of physical laboratory performance and does not replace the standard verification of M_{Rd}.

In the approved a priori calculation, the compressed zone of concrete is described as an equivalent rectangular block with parameters η = 1.0 and λ = 0.8. The resultant force in the concrete is $C_c = \eta \cdot f_c \cdot b \cdot \lambda_x$, and its centre of application is situated at a distance of $\lambda_x/2$ from the compressed face. The strain in the upper steel is defined as $\varepsilon'_s = \varepsilon_{cu}(x-d')/x$, its stress as $\sigma'_s = \min(E_s \cdot \varepsilon'_s; 240\ \text{MPa})$, and the force as $C'_s = A'_s\sigma'_s$. The tensile force in the GFRP is $T_f = A_f \cdot \sigma_f$. The depth of the neutral axis x is found as the root of the equilibrium equation $T_f - C_c - C'_s = 0$ in the interval $d' < x < h_0$; to obtain tabulated values, the interval is narrowed iteratively until the change in x is no more than 0.01 mm. Once x has been determined, the value $M_{u,pred}$is calculated using equation (10).

$$\varepsilon_{fu} = f_{fu}/E_f, \tag{6}$$

where ε_{fu} is the ultimate relative strain of the GFRP reinforcement based on its specified strength;
f_{fu} – the nominal tensile strength of the GFRP;
E_f – the specified modulus of elasticity of the GFRP.

$$\varepsilon_f = \varepsilon_{cu}(h_0 - x)/x\ , \tag{7}$$

where ε_f – the design relative strain of the GFRP in the ultimate limit state;
ε_{cu} – ultimate strain of compressed concrete;
h_0 – effective height of the cross-section;
x – the depth of the compressed zone and the position of the neutral axis relative to the compressed face.

$$\sigma_f = min(E_f\varepsilon_f;\ f_{fu}), \tag{8}$$

where σ_f – stress in the GFRP;
E_f – the modulus of elasticity of the GFRP;
ε_f – its relative strain;
f_{fu} – the specified tensile strength of the GFRP.

$$T_f = C_c + C'_s, \tag{9}$$

where T_f – tensile force in the GFRP;
C_c – the resultant compressive force in the concrete;
C'_s – the force in the upper longitudinal reinforcement. The equation expresses the equilibrium of the cross-section.

$$M_{u,pred} = C_c(h_0 - \lambda_x/2) + C'_s(h_0 - d'), \tag{10}$$

where $M_{u,pred}$ – the a priori predicted ultimate flexural capacity;
C_c – resultant force in the concrete;
C'_s – force in the upper steel;
h_0 – effective height;
$\lambda_x/2$ – distance from the compressed face to the resultant force in the concrete;
d′ – distance from the compressed face to the centre of the top reinforcement.

To predict laboratory failure, the above set of a priori concrete characteristics, the specified characteristics of GFRP and the parameters of the upper steel are used without the reliability reduction factors characteristic of M_{Rd}. The calculated values of x, ε_f, $M_{u,pred}$ and $P_{u,pred}$ for the nine combinations are given in Table 4. For all nine combinations, $\varepsilon_f < \varepsilon_{fu}$ is calculated; therefore, it is predicted that the limit state of the compressed concrete will be reached before the GFRP fails. This is a priori calculated prediction, not an experimental result.

Table 4 – A priori prediction for nine cross-sections

GFRP	Concrete	x, mm	ε_f , ‰	$M_{u,pred}$, kN·m	$P_{u,pred}$, kN
3Ø8	C16/20	53.41	11.98	18.38	88.6
3Ø8	C20/25	47.43	12.95	20.07	96.7
3Ø8	C25/30	42.58	13.71	21.42	103.2
3Ø12	C16/20	76.24	7.28	24.07	116.0
3Ø12	C20/25	68.23	7.95	26.67	128.5
3Ø12	C25/30	61.64	8.48	28.80	138.8
2Ø18	C16/20	89.81	5.61	27.17	130.9
2Ø18	C20/25	80.82	6.17	30.37	146.4
2Ø18	C25/30	73.33	6.62	33.02	159.1

The support axes are situated 100 mm from the ends, so the span L = 980 mm. The distance between the two forces is 150 mm, the lever arm a = 415 mm, and the constant moment zone is 150 mm. The total force P is transmitted by a DG-50 hydraulic jack via a symmetrical rigid cross-beam; the equality of the two forces is ensured geometrically and is not monitored by separate sensors.

$$M = \frac{Pa}{2} = 0{,}2075P, \quad (11)$$

where M is the bending moment in the constant moment zone; P is the total force from the jack; a = 415 mm – the lever arm between the line of action of the total force and the support in the adopted four-point diagram.

The load is set in step-by-step force control mode. The basic increment is 1 tonne-force = 9.80665 kN. The force is monitored using an MTI-1216 pressure gauge (0–160 kgf/cm², accuracy class 0.6%) in accordance with the working calibration curve of the 'jack–pressure gauge' system installed on the test press. If n is the number of scale divisions from the zero reading, the working ratio is P ≈ n/4 tonne-force, or P ≈ 2.4517n kN; thus, 4 divisions correspond approximately to 1 ts. This relationship is not regarded as a certified calibration of the entire measuring circuit. The theoretical values P_{min} and P_{max} are target values; in practice, the actual reading recorded on the pressure gauge is used. No specific historical rule for rounding the target force to a particular scale division is recorded in the primary documentation, nor can it be reconstructed retrospectively. A constant rate of transition between stages is not specified.

One cycle is defined as the sequence 'lower limit → upper limit → lower limit' with prescribed 10-minute dwell times at the upper and lower limits; the cycle is considered complete upon the expiry of the dwell time at its lower limit. For $X_3 = -1$, 10 cycles of $0 \leftrightarrow 0.4\ M_{u,pred}$ are performed. For $X_3=0$, after an initial adjustment to $0.4\ M_{u,pred}$ and a 10-minute dwell period – which are not counted as a cycle – 10 cycles of $0.4 \leftrightarrow 0.6 M_{u,pre}$ are performed. For $X_3 = +1$, after an initial adjustment to $0.6\ M_{u,pred}$ and a 10-minute dwell period, 10 cycles of $0.6 \leftrightarrow 0.8\ M_{u,pred}$ are performed. DSTU-N B V.2.6-185:2012 is used for static design assumptions, but not as a normative basis for the ten-cycle protocol, as its scope of application does not cover endurance calculations [20]. Upon completion of the lower load stage of the tenth cycle, a separate full unloading to zero is performed for $X_3 = 0$ and $X_3 = +1$; for $X_3 = -1$, the lower limit of the tenth cycle is already zero.

The theoretical target limits for cyclic loading at each point in the matrix are given in Table 5.

Note to Table 5. Values in kN and ts are given to the accuracy specified in the approved experimental protocol. Due to independent rounding of the values in kN and ts, they should not be used for mutual conversion to the last digit; when processing the results, the actual load value achieved is used.

The design verification of inclined cross-sections is carried out taking into account double-strand Ø6 A240 bars with a spacing of 50 mm in accordance with the regulatory provisions [23]. A basic verification of the anchoring of straight GFRP bars with an available length of 515 mm shows the limit case for 2Ø18/C16/20; therefore, any possible anchorage, shear or local contact failure is recorded as a non-target failure mode and is not reclassified as a bending failure. If a non-target mechanism occurs prior to the planned ultimate bending state, the corresponding data point is retained in the descriptive dataset with the actual failure mode, but is considered censored with respect to the

post-cyclic ultimate flexural capacity of the normal cross-section and is not included in the regression model for this response. An automatic repeat specimen is not part of the approved 15-point programme; any confirmatory repeat is recorded as a separate additional test.

Table 5 – Theoretical target cyclic levels

Beam	**X_3**	**$M_{u,pred}$, kN·m**	**P_{min}– P_{max}, kN**	**P_{min} –P_{max}, tf**
B1	+1	33.02	95.46–127.28	9.74–12.98
B2	−1	33.02	0–63.64	0–6.49
B3	+1	27.17	78.54–104.72	8.01–10.68
B4	+1	21.42	61.92–82.56	6.31–8.42
B5	−1	27.17	0–52.36	0–5.34
B6	−1	21.42	0–41.28	0–4.21
B7	+1	18.38	53.16–70.88	5.42–7.22
B8	−1	18.38	0–35.44	0–3.61
B9	0	30.37	58.56–87.84	5.97–8.95
B10	0	20.07	38.68–58.02	3.94–5.92
B11	0	28.80	55.52–83.28	5.66–8.49
B12	0	24.07	46.40–69.60	4.73–7.10
B13	+1	26.67	77.10–102.80	7.86–10.49
B14	−1	26.67	0–51.40	0–5.24
B15	0	26.67	51.40–77.10	5.24–7.86

Relative deformations are recorded by a modular multi-channel data acquisition system (MMCDAS ‘Om-Technologia’) with an actual sampling interval of 16 ms, i.e. 62.5 Hz. Seven sensors are installed on the concrete: one at the centre of the top face, five longitudinally oriented on the side surface at levels y = 25; 75; 125; 175; 225 mm from the top face, and one at the centre of the bottom face. For concrete, the confirmed sensor model is Zhuoye BX120-50AA, with a 50 mm base. On each GFRP rod, two diametrically opposed strain gauges with a 10 mm base are installed in the central cross-section; their manufacturer’s code is Zhuoye BX120-8AA, with a 10 mm base.

Deflection at the geometric centre of the beam is determined using an Aistov 6PAO deflectometer immediately upon reaching the upper or lower limit. A repeat manual reading at the end of the 10-minute holding period is not carried out. Settlement of the supports is not measured separately. On one side face, the maximum visually detected width of a flexural crack is checked using a template with 0.1 mm graduations. The methodology does not provide for the mapping, numbering, total count or spacing of cracks.

Research results. The main outcome of this work is not an experimental relationship, but a developed methodological system combining a 15-point matrix based on FCCD geometry, an a priori $M_{u,pred}$ for cycle normalisation, three ordered modes X_3, a single four-point scheme, continuous strain gauge measurement, predefined processing algorithms, post-cycle loading to failure, and explicit statistical constraints. The difference from [13] lies not in the number of beams, but in the physical structure of the factors, the method of normalising cyclic loading, the measurement system and the rules of interpretation, which are summarised in Table 1.

$$\bar{\varepsilon}_{f,j} = (\varepsilon_{f,j(6)} + \varepsilon_{f,j(12)})/2, \quad (12)$$

where $\bar{\varepsilon}_{f,j}$ is the average strain of the jth GFRP rod; $\varepsilon_{f,j(6)}$ and $\varepsilon_{f,j(12)}$ – the readings of two diametrically opposed strain gauges on this rod.

$$\varepsilon_c(y) = \varepsilon_0 + \kappa_y, \quad (13)$$

where $\varepsilon_{c(y)}$ is the strain in the concrete at level y;

ε_0 – the extrapolated strain at y = 0;

κ – the curvature of the cross-section;

y – the height coordinate of the cross-section, measured downwards from the top face.

$$y_{NA} = -\varepsilon_0/\kappa, \quad (14)$$

where y_{NA} is the position of the neutral axis relative to the top face; ε_0 and κ are the parameters of the linear approximation of the strain profile $\varepsilon(y)$. The formula is applicable only provided that zero strain is correctly identified.

$$Ki = (P_{max,i} - P_{min,i})/(\delta_{max,i} - \delta_{min,i}), \quad (15)$$

where K_i is the cyclic secant stiffness of the i-th cycle; $P_{max,i}$ and $P_{min,i}$ are the actual maximum and minimum cycle forces; $\delta_{max,i}$ and $\delta_{min,i}$ are the corresponding deflections recorded immediately after the cycle limits are reached.

$$\eta_{K,i} = Ki/K1, \quad (16)$$

where $\eta_{K,i}$ is the cyclic stiffness retention ratio; K_i – the stiffness of the i-th cycle; K1 – the stiffness of the first cycle of the corresponding mode.

$$D_{K,i} = 1 - \eta_{K,i}, \quad (17)$$

where $D_{K,i}$ – relative cyclic stiffness degradation; $\eta_{K,i}$ – stiffness retention ratio of the i-th cycle.

To process the lateral strain gauge series, the y-coordinate is measured downwards from the top face; tensile strains are taken as positive, and compressive strains as negative. For each reference state, the relationship $\varepsilon_c(y) = \varepsilon_0 + \kappa y$ is determined using the ordinary least squares method based on valid lateral channels. At least four out of the five lateral sensors are used to calculate κ and y. A channel is excluded only for a predefined and documented reason: visible intersection of the measuring base by a crack, signal loss or saturation, or an obvious electrical fault. For each approximation, a list of included channels, the coefficient of determination R^2 and the regression residuals are recorded. A fixed universal R^2 threshold is not set, so as not to introduce an arbitrary criterion following crack formation; interpretation of κ and y is permitted only in the absence of systematic curvature in the residuals and provided the sign profile is physically consistent. If there are fewer than four valid channels, if the estimate of κ is numerically indistinguishable from zero within the regression accuracy, or if zero strain cannot be reliably localised, y is not determined for that state.

Upon completion of the tenth cycle, the beam is completely unloaded, the immediate residual state is recorded, and it is reloaded from zero in increments of 1 tonne-force until physical failure. For $X_3 = 0$ and +1, no additional 10-minute dwell at zero is applied; for $X_3 = -1$, the zero dwell is the standard lower limit of the tenth cycle.

$$M_{u,exp} = 0{,}2075 P_{u,exp}, \quad (18)$$

where $M_{u,exp}$ – the experimentally determined post-cycle ultimate bending moment; $P_{u,exp}$ – the actual recorded ultimate force of the jack during the post-cycle loading stage.

$$\psi_M = M_{u,exp}/M_{u,pred}, \quad (19)$$

where ψ_M – the ratio of the post-cycle experimental ultimate flexural capacity to its a priori prediction; $M_{u,exp}$ – the experimental moment; $M_{u,pred}$ – the a priori predicted moment.

The value ψ_M is interpreted as the ratio of the post-cycle experimental ultimate flexural capacity to the a priori predicted ultimate bending capacity, but not as a pure coefficient of strength loss due to cycling, since there is no identical monotonic reference beam for each combination of factors.

The statistical analysis begins with direct comparisons of the combinations actually implemented. The most informative axial triplets are B−100–B000–B100, B0−10–B000–B010 and B00−1–B000–B001, which vary one factor whilst keeping the other two constant. In quantitative regression, X_1 is represented by the actual value μ_f, X_2 as a three-level factor for concrete class C16/20, C20/25, C25/30, and X_3 as a three-level categorical factor with a natural order. Due to the absence of a precise ‘beam–batch’ relationship, the actual strength of the control specimens is not used as an individual continuous covariate for a single beam. The average prism value of the three control prisms of the corresponding class may only be used as a descriptive characteristic of the class and for a separate sensitivity check of the a priori calculation. The full quadratic model with X_3^2 is not adopted as the primary model, since X_3 simultaneously affects M_{min}, M_{max}, M_m, M_a and R.

Due to the absence of replicates and the presence of a single central point, it is not possible to independently estimate the pure error or perform a formal lack-of-fit test. Therefore, the matrix is suitable for direct comparisons, trend analysis and compact exploratory modelling, but not for claiming a fully statistically validated quadratic surface or a confirmed optimum [14–16].

The limitations of the methodology also include: the absence of an independent load cell and

separate monitoring of the two forces; non-standardised transition speed between steps; the absence of a documented rule for rounding the theoretical set-point to the pressure gauge scale; the absence of measurements of support settlement; manual inspection for cracks on only one face; the absence of complete metrological documentation for part of the strain gauge circuit and the technical specifications of 6PAO; the use of GFRP technical specifications without a recorded batch passport number; the absence of documented randomisation and a precise link between each beam and a specific concrete mix. These limitations define the scope of interpretation and cannot be replaced by assumptions or the reconstruction of missing data.

Conclusions. A formalised methodology has been developed for low-cycle testing of concrete beams with GFRP reinforcement in the tension zone, which combines the a priori determination of $M_{u,pred}$, cycle normalisation, the recording of stress–strain state parameters, the determination of the residual state, and post-cycle loading until physical failure. A 15-point three-factor matrix based on FCCD geometry has been established: 8 corner points, 6 axial points and 1 central point. The factors are the actual level of GFRP reinforcement X_1, the concrete class X_2 and the ordered low-cycle loading regime X_3; the latter is not treated as a conventional continuous physical variable. For nine combinations of reinforcement and concrete, an a priori design basis $M_{u,pred}$ = 18.38–33.02 kN·m has been established. This is used to specify the cycles; $M_{u,pred}$ is not M_{cr}, M_{Rd} or an experimentally determined ultimate moment. The measurement system and predefined processing rules enable the subsequent determination of deflections, strains of the GFRP and concrete, the ε(y) profile, curvature κ, the position of the neutral axis, the maximum width of the flexural crack, K_i, $\eta_{K,i}$, $D_{K,i}$ and the post-cycle $P_{u,exp}$ and $M_{u,exp}$ within the limits of the actually available channels. Due to the single central point and the absence of independent replicates, the pure error and the lack-of-fit test are not available; the matrix results should be used for direct comparisons, trend analysis and exploratory modelling without claiming a fully validated quadratic surface or a confirmed optimum. The scientific and methodological novelty lies in the integration, within a single programme, of an FCCD-geometry matrix, a composite factor X_3, a priori normalisation according to $M_{u,pred}$, through-depth strain monitoring of the central cross-section, cyclic stiffness indices and a separate post-cyclic ultimate stage. The practical significance lies in the formalisation of a protocol that establishes uniform rules for loading, measurement and interpretation for all 15 points, whilst explicitly defining the limits of the validity of the data obtained. Further development of this work involves applying the established methodology to actual experimental data, comparing the post-cyclic ultimate flexural capacity with the a priori prediction, verifying trends based on X_1–X_3 and, where possible, conducting independent confirmatory tests to assess inter-specimen variability.

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ТРИФАКТОРНА ЕКСПЕРИМЕНТАЛЬНА МАТРИЦЯ ТА МЕТОДИКА МАЛОЦИКЛОВИХ ВИПРОБУВАНЬ БЕТОННИХ БАЛОК ІЗ РОБОЧОЮ GFRP-АРМАТУРОЮ В РОЗТЯГНУТІЙ ЗОНІ

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Анотація. Методична неоднорідність опублікованих програм повторного навантаження GFRP-армованих згинальних елементів ускладнює пряме зіставлення даних, оскільки в них відрізняються межі циклу, кількість повторів, спосіб нормування навантаження та набір контрольованих показників. Метою роботи є розроблення та методичне обґрунтування формалізованої програми експериментального дослідження бетонних балок із робочою поздовжньою GFRP-арматурою в розтягнутій зоні за впорядкованих режимів малоциклового навантаження. Робота має методичний характер; її науковим результатом є експериментальна процедура, а не кількісні результати випробувань серії балок. Програма охоплює 15 зразків і три фактори: рівень робочого GFRP-армування X_1, клас бетону X_2 та впорядкований режим малоциклового навантаження X_3. Матрицю сформовано за геометрією гранецентрованого центрального композиційного плану: вісім кутових, шість осьових і одна центральна точка. Для X_1 прийнято схеми 3Ø8, 3Ø12 і 2Ø18 GFRP з фактичними значеннями $\mu_f=A_f/(bh)$ 0,503; 1,130 і 1,697 %, для X_2 – класи C16/20, C20/25 та C25/30. X_3 трактовано як комплексний режим, оскільки між його рівнями одночасно змінюються M_{min}, M_{max}, середній момент, амплітуда та R. Циклічну дію нормують відносно апріорної прогнозованої лабораторної граничної згинальної здатності $M_{u,pred}$, яку визначають за сумісністю деформацій та рівновагою нормального перерізу. Сформовано режими $0\leftrightarrow0{,}4M_{u,pred}$, $0{,}4\leftrightarrow0{,}6M_{u,pred}$ і $0{,}6\leftrightarrow0{,}8M_{u,pred}$, кожний із десятьма циклами та 10-хвилинними витримками на верхній і нижній межах. Методика передбачає чотириточковий згин при прольоті 980 мм, плечі 415 мм і зоні постійного моменту 150 мм; зв’язок між сумарною силою домкрата та моментом становить $M=0{,}2075P$. Система вимірювань включає центральний прогин, безперервну реєстрацію деформацій бетону та GFRP із частотою 62,5 Гц і ручне визначення максимальної ширини розкриття нормальної тріщини. Наперед задано алгоритми визначення усередненої деформації GFRP, профілю $\varepsilon(y)$, кривизни, положення нейтральної осі та циклічної секантної жорсткості. Через відсутність повторів і наявність лише однієї центральної точки матриця призначена для прямих порівнянь, аналізу тенденцій і пошукового моделювання без формального тесту невідповідності. Науково-методична новизна полягає в інтеграції єдиної факторної матриці, комплексно заданих циклічних режимів, наскрізної тензометрії та наперед визначених правил обробки в одному формалізованому протоколі.

Ключові слова: склопластикова арматура, GFRP, бетонна балка, малоциклове навантаження, експериментальна матриця, прогин, деформації, циклічна жорсткість.

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